Problem of the Week
Problem C and Solution
A Few More Marbles

Problem
Mark has a bag that contains exactly 3 black marbles, 6 gold marbles, 2 purple marbles and 6 red marbles. Mark finds a number of white marbles and adds them to the bag. He tells Sam that if she now draws a marble at random from the bag, the probability of it being black or gold is $\frac{3}{7}$. How many white marbles did Mark add to the bag?

Solution
In order to determine the probability that a marble drawn from the bag is black or gold, we divide the number of black and gold marbles in the bag by the total number of marbles in the bag. In other words,

$$\text{Probability of selecting a black or gold marble} = \frac{\text{Number of black or gold marbles}}{\text{Total number of marbles}}$$

When Mark adds white marbles to the bag, this does not change the number of black or gold marbles. Therefore, the number of black or gold marbles in the bag is $3 + 6 = 9$.

We are also given that the probability of drawing a black or gold marble is $\frac{3}{7}$.

So the equation

$$\text{Probability of selecting a black or gold marble} = \frac{\text{Number of black or gold marbles}}{\text{Total number of marbles}}$$

becomes

$$\frac{3}{7} = \frac{9}{\text{Total number of marbles}}$$

Since $\frac{3}{7} = \frac{9}{21}$, this tells us

$$\frac{9}{21} = \frac{9}{\text{Total number of marbles}}$$

Therefore, the total number of marbles is 21.

Originally, there were $3 + 6 + 2 + 6 = 17$ marbles in the bag. Then Mark added some white marbles. Since the total number of marbles in the bag after adding some white marbles is 21, Mark must have added $21 - 17 = 4$ white marbles.