



Problem of the Week

Problem C and Solution

Mystery Number

Problem

A positive integer has exactly eight positive divisors. If two of the divisors are 21 and 35, what is the number?

Solution

Let n represent the number we are looking for.

We know that four of the positive divisors of n are 1, 21, 35 and n . In our solution we will first find the remaining four positive divisors and then determine n .

Since 21 is a divisor of n and $21 = 3 \times 7$, 3 and 7 must also be divisors of n .

Since 35 is a divisor of n and $35 = 5 \times 7$, 5 must also be a divisor of n .

Since 3 is a divisor of n and 5 is a divisor of n , and since 3 and 5 have no common divisors, $3 \times 5 = 15$ must also be a divisor n .

We have found all eight of the positive divisors of the unknown number. The positive divisors are 1, 3, 5, 7, 15, 21, 35 and n . We now need to determine n .

From the list of divisors, we see that the prime factors of n are 3, 5 and 7, and it follows that $n = 3 \times 5 \times 7 = 105$.

Therefore, the number is 105.

