Problem of the Week
Problem D and Solution
Select Addresses

Problem

A hardware store sells single digits to be used for house numbers. There are five 5s, four 4s, three 3s, and two 2s available. From this selection of digits, a customer is able to purchase her three-digit house number. Determine the number of possible three-digit house numbers this customer could form.

Solution

Solution 1

Let's suppose that there are three or more 2s available.
For the first digit, the customer can choose from the digits 5, 4, 3, and 2. Therefore, there are 4 choices for the first digit.
Similarly, there are 4 choices for the second digit and 4 choices for the third digit.
This gives $4 \times 4 \times 4 = 64$ possible three-digit house numbers that can be made.
But there are actually only two 2s available, so not all of these house numbers can be made. In particular, the house number 222 cannot be made, but all others can.
Therefore, the customer could form $64 - 1 = 63$ different three-digit house numbers.

Solution 2

Let's look at the different cases.

Case 1: All three digits in the house number are the same
The house number could then be 555, 444 or 333. The customer cannot form 222 since only two 2s are available. Therefore, there are 3 three-digit house numbers with all three digits the same.

Case 2: Two digits are the same and the third digit is different
There are four choices for the digits that are the same, namely 5, 4, 3, and 2. For each of these possible choices, there are 3 choices for the third different digit. For example, if two of the digits are 5s, then the third digit could be 4, 3 or 2. Therefore, there are $4 \times 3 = 12$ ways to choose the digits. For each of these choices, there are 3 ways to arrange the digits. For example, suppose the digits are a, a and b. The house number could be aab, aba or baa. Therefore, there are $12 \times 3 = 36$ three-digit house numbers with two digits the same and one different.

Case 3: All three digits are different
The customer has 4 choices for the first digit (namely, 5, 4, 3, or 2). Once that digit is chosen, there are 3 choices for the second digit. Once the first and second digits are chosen, there are 2 choices for the third digit. Therefore, there are $4 \times 3 \times 2 = 24$ three-digit house numbers with all three digits different.

Therefore, the total number of three-digit house numbers that the customer can form is $3 + 36 + 24 = 63$.