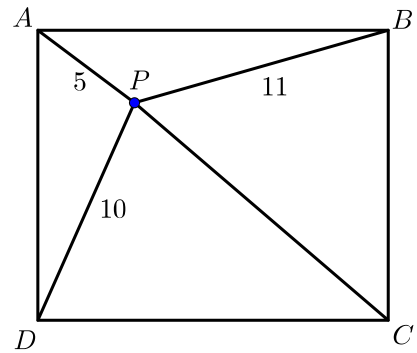
Problem of the Week

Problem D and Solution

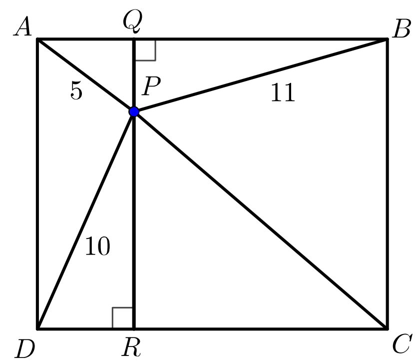
From the Four Corners



Problem

In the diagram, $ABCD$ is a rectangle. Point P is located inside the rectangle so that the distance from P to A is 5 cm, the distance from P to B is 11 cm, and the distance from P to D is 10 cm. How far is P from C?

Solution



We start by drawing a perpendicular from P to AB. Let Q be the point of intersection. Let's draw another perpendicular from P to DC. Let R be the point of intersection.

Since QP is perpendicular to AB, $\angle AQP = 90^\circ$ and $\angle BQP = 90^\circ$. Since PR is perpendicular to DC, $\angle DRP = 90^\circ$ and $\angle CRP = 90^\circ$. We also have that $AQ = DR$ and $BQ = CR$.

We can apply the Pythagorean Theorem in $\triangle AQP$ and $\triangle BQP$.
From $\triangle AQP$ we have $AQ^2 + QP^2 = AP^2$, and so $AQ^2 + QP^2 = 5^2 = 25$.
Rearranging, we have $QP^2 = 25 - AQ^2$ (1).

From $\triangle BQP$ we have $BQ^2 + QP^2 = BP^2$, and so $BQ^2 + QP^2 = 11^2 = 121$.

Rearranging, we have $QP^2 = 121 - BQ^2$ (2).

Since $QP^2 = QP^2$, from (1) and (2) we find that $25 - AQ^2 = 121 - BQ^2$ or $BQ^2 - AQ^2 = 96$.
Since $AQ = DR$ and $BQ = CR$, this also tells us $CR^2 - DR^2 = 96$ (3).

We can now apply the Pythagorean Theorem in $\triangle DRP$ and $\triangle CRP$.
From $\triangle DRP$ we have $DR^2 + RP^2 = DP^2$, and so $DR^2 + RP^2 = 10^2 = 100$.
Rearranging, we have $RP^2 = 100 - DR^2$ (4).

When we apply the Pythagorean Theorem to $\triangle CRP$ we have $CR^2 + RP^2 = CP^2$.
Rearranging, we have $RP^2 = CP^2 - CR^2$ (5).

Since $RP^2 = RP^2$, from (4) and (5) we find that $100 - DR^2 = CP^2 - CR^2$, or $CR^2 - DR^2 = CP^2 - 100$ (6).

From (3), we have $CR^2 - DR^2 = 96$, so (6) becomes $96 = CP^2 - 100$ or $CP^2 = 196$.
Thus $CP = 14$, since $CP > 0$.

Therefore the distance from P to C is 14 cm.