



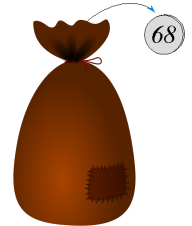
## Problem of the Week

### Problem D and Solution

#### Taken Token

#### Problem

Some tokens are placed in a bag. Each of the tokens has a positive integer stamped on one of its sides. It is possible that more than one token in the bag has the same number stamped on it. The average of all of the numbers stamped on the tokens in the bag is 56. If a token with the number 68 on it is removed from the bag, the average of the numbers stamped on the remaining tokens is 55. Determine the largest possible integer that could appear on one of the tokens in the bag.



#### Solution

To calculate an average, determine the sum of the numbers in the set and divide by the number of numbers in the set. It follows that the sum of the numbers in a set is the average times the number of numbers in the set.

Let  $n$  represent the number of tokens in the bag.

The total of the numbers on all of the tokens in the bag is  $56n$ . After the token with the number 68 stamped on it is removed, the total of the numbers on the remaining tokens is  $(56n - 68)$  and there are  $(n - 1)$  tokens remaining in the bag. The average of the numbers stamped on the tokens remaining in the bag is 55 so it follows that

$$\begin{aligned}\frac{56n - 68}{n - 1} &= 55 \\ 56n - 68 &= 55(n - 1) \\ 56n - 68 &= 55n - 55 \\ n &= 13\end{aligned}$$

Since  $n = 13$ , there were originally 13 tokens in the bag and the total of the numbers stamped on the tokens in the bag was  $56n = 56(13) = 728$ .

To determine the highest possible value on a token, stamp eleven of the tokens with the smallest possible positive integer, namely 1. The twelfth token has the number 68 stamped on it. The largest possible value that could be stamped on the remaining token is,  $728 - 11 \times 1 - 68 = 649$ .

As an extension, consider how the answer would change if no two tokens were stamped with the same number.

