Problem of the Week
Problem E and Solution
Measure Twice, Cut Once

Problem
You are given 100 cm of rope and are asked to cut the rope exactly once so that:

1. You have two pieces.
2. The first piece is used to form a rectangle with one side 8 cm long.
3. The second piece is used to form a square.
4. The area of the square and the area of the rectangle are equal.

Where should the cut be made?

Solution
Let the length of the piece of rope used to form the square be $4x$ cm. This would be the perimeter of the square. Then the side length of the square would be $4x ÷ 4 = x$ cm. The area of the square is $(x)(x) = (x^2)$ cm$^2$. (1)

The length of the piece of rope used to form the rectangle is $(100 - 4x)$ cm. This would be the perimeter of the rectangle. If the width of the rectangle is 8 cm, then there are $100 - 4x - 8 - 8 = (84 - 4x)$ cm left to form the lengths of the two other sides of the rectangle. Therefore, the length of the rectangle is $\frac{84-4x}{2} = (42 - 2x)$ cm. The area of the rectangle is $(8)(42 - 2x) = (336 - 16x)$ cm$^2$. (2)

But the area of the square equals the area of the rectangle, so by equating (1) and (2) we obtain:

$$x^2 = 336 - 16x$$
$$x^2 + 16x - 336 = 0$$
$$(x - 12)(x + 28) = 0$$
$$x = 12 \text{ or } x = -28$$

Since $x$ is the length of the side of the square, $x = -28$ is inadmissible. Therefore, $x = 12$ cm. The area of the square is $12 \times 12 = 144$ cm$^2$. The length of the rectangle is $42 - 2x = 42 - 24 = 18$ cm. The area of the rectangle is $18 \times 8 = 144$ cm$^2$. (These calculations were not required but are provided as a check of the correctness of the result.)

Then $4x = 4(12) = 48$ cm. The cut should be made 48 cm from one end creating a 52 cm piece for the rectangle and a 48 cm piece for the square.