Problem of the Week
Problem C and Solution
Top Triangle

Problem
The area of $\triangle ACD$ is twice the area of square $BCDE$. $AC$ and $AD$ meet $BE$ at $K$ and $L$ respectively such that $KL = 6$ cm.

If the side length of the square is 8 cm, determine the area of the top triangle, $\triangle AKL$.

Solution
Solution 1
In the first solution we will find the area of square $BCDE$, the area of $\triangle ACD$, the area of trapezoid $KCDL$, and then the area of $\triangle AKL$.

To find the area of a square, multiply the length times the width. To find the area of a trapezoid, multiply the sum of the lengths of the two parallel sides by the height and divide the product by 2.

\[
\text{Area of square } BCDE = 8 \times 8 = 64 \text{ cm}^2
\]
\[
\text{Area } \triangle ACD = 2 \times \text{Area of Square } BCDE = 2 \times 64 = 128 \text{ cm}^2
\]

In trapezoid $KCDL$, the two parallel sides are $KL$ and $CD$, and the height is the width of square $BCDE$, namely $BC$.

\[
\text{Area of trapezoid } KCDL = (KL + CD) \times BC \div 2 = (6 + 8) \times 8 \div 2 = 14 \times 8 \div 2 = 56 \text{ cm}^2
\]
\[
\text{Area } \triangle AKL = \text{Area } \triangle ACD - \text{Area of trapezoid } KCDL = 128 - 56 = 72 \text{ cm}^2
\]

Therefore, the area of $\triangle AKL$ is 72 cm$^2$. 
Solution 2

Construct the altitude of \( \triangle ACD \) intersecting \( BE \) at \( P \) and \( CD \) at \( Q \). In this solution we will find the height of \( \triangle AKL \) and then use the formula for the area of a triangle to find the required area.

To find the area of a square, multiply the length times the width. To find the area of a triangle, multiply the length of the base times the height and divide the product by 2.

\[
\text{Area of square } BCDE = 8 \times 8
\]
\[
= 64 \text{ cm}^2
\]
\[
\text{Area } \triangle ACD = 2 \times \text{Area of Square } BCDE
\]
\[
= 2 \times 64
\]
\[
= 128 \text{ cm}^2
\]

But Area \( \triangle ACD = CD \times AQ \div 2 \)
\[
128 = 8 \times AQ \div 2
\]
\[
128 = 4 \times AQ
\]
\[
\therefore AQ = 32 \text{ cm}
\]

We know that \( AQ = AP + PQ \), \( AQ = 32 \text{ cm} \) and \( PQ = 8 \text{ cm} \), the side length of the square. It follows that \( AP = AQ - PQ = 32 - 8 = 24 \text{ cm} \).

\[
\therefore \text{Area } \triangle AKL = KL \times AP \div 2
\]
\[
= 6 \times 24 \div 2
\]
\[
= 72 \text{ cm}^2
\]

Therefore, the area of \( \triangle AKL \) is 72 cm\(^2\).