Problem of the Week
Problem D and Solution
Spare Change

Problem
Kees emptied his piggy bank of all its 34 coins with a total value of $5.30. The coins are nickels, dimes or quarters only. There are twice as many quarters as dimes. How many of each type of coin does Kees have?

Solution
Let \( n \) be the number of nickels, \( d \) be the number of dimes and \( q \) be the number of quarters.

From the total number of coins we get the equation \( n + d + q = 34 \) \( \text{(1)} \).

From the value of the coins we get the equation \( 5n + 10d + 25q = 530 \) \( \text{(2)} \).

We also know that \( q = 2d \) \( \text{(3)} \).

Substituting equation \( (3) \) into equation \( (1) \) and simplifying

\[
\begin{align*}
n + d + 2d &= 34 \\
n + 3d &= 34 \quad (4)
\end{align*}
\]

Substituting equation \( (3) \) into equation \( (2) \) and simplifying

\[
\begin{align*}
5n + 10d + 25(2d) &= 530 \\
5n + 60d &= 530 \\
n + 12d &= 106 \quad (5)
\end{align*}
\]

We can isolate \( n \) in equation \( (4) \) to get \( n = 34 - 3d \).

We can isolate \( n \) in equation \( (5) \) to get \( n = 106 - 12d \).

We equate the two \( n \)'s and solve for \( d \):

\[
\begin{align*}
34 - 3d &= 106 - 12d \\
-3d + 12d &= 106 - 34 \\
9d &= 72 \\
d &= 8
\end{align*}
\]

We now substitute \( d = 8 \) into equation \( (4) \) to solve for \( n \):

\[
\begin{align*}
n + 3d &= 34 \\
n + 3(8) &= 34 \\
n + 24 &= 34 \\
n &= 10
\end{align*}
\]

Finally, substitute \( d = 8 \) into equation \( (3) \) to find \( q = 2(8) = 16 \).

Therefore, Kees has 10 nickels, 8 dimes and 16 quarters.