Problem of the Week

Problem E

Four Tokens Please

In a game, players compete to earn points. The banker “pays” the players using tokens. A circular token is worth 1 point, a square token is worth 5 points, a triangular token is worth 10 points, and a hexagonal token is worth 25 points.

When paying a player, the banker must follow the minimum token rule. That is, the banker must pay using the least number of tokens possible. For example, to pay a player 30 points, there are many combinations of tokens that work. The banker could use 30 circular tokens \((30 \times 1 \text{ point} = 30 \text{ points})\) or 2 square tokens and 2 triangular tokens \((2 \times 5 \text{ points} + 2 \times 10 \text{ points} = 30 \text{ points})\), a total of four tokens. There are other possible ways to combine tokens to obtain 30 points. However, since the banker must pay using the minimum token rule, the banker would pay using 2 tokens, 1 hexagonal token and 1 square token \((1 \times 25 \text{ points} + 1 \times 5 \text{ points} = 30 \text{ points})\).

How many different point totals can the banker generate using exactly four tokens, provided the banker follows the minimum token rule? For example, a total of 41 points can be generated using exactly four tokens since this is the minimum number of tokens required to produce that total. A total of 30 points would not be generated using exactly four tokens since this total can be produced using fewer tokens.