Problem of the Week
Problem E and Solution
What’s Left?

Problem

$BCDE$ is a square with sides of length 20 cm. $BE$ is extended to $A$ such that the area of $\triangle ABC$ is twice the area of the square. The figure $ABCD$ is enclosed in a circle with diameter $AC$ and point $B$ on the circumference of the circle. See the diagram to the right. Determine the area inside the circle but outside figure $ABCDF$.

Solution

To find the area of the unshaded region, we need to find the area of the circle and subtract the area of the shaded figure $ABCDF$. To find the area of the circle we need the radius, which is half the length of diameter $AC$. To find the area of the shaded figure, we need to find the areas of square $BCDE$ and $\triangle AEF$. We will need to find the length of $EF$.

Area of Square $BCDE = 20 \times 20 = 400$ cm$^2$

Area of $\triangle ABC = 2 \times $ Area of Square $BCDE = 800$ cm$^2$

But Area of $\triangle ABC = (BC)(AB) \div 2$

$\therefore 800 = (20)(AB) \div 2$

$AB = 80$ cm

Then $AE = AB - BE = 80 - 20 = 60$ cm.

Since $\angle AEF = \angle ABC = 90^o$ and $\angle FAE = \angle CAB$, then $\triangle AEF \sim \triangle ABC$.

$\therefore \frac{AE}{AB} = \frac{EF}{BC}$

$\frac{60}{80} = \frac{EF}{20}$

$EF = 15$ cm

Since $\triangle ABC$ is right angled, $AC^2 = BC^2 + AB^2$

$= 20^2 + 80^2$

$= 6800$

$AC = 20\sqrt{17}$ cm

But $AC$ is the diameter of the circle so the radius is $10\sqrt{17}$.

Unshaded Area = Area of Circle – Area of $ABCDF$

= Area of Circle – (Area of Square $BCDE$ + Area of $\triangle AEF$)

= $\pi(10\sqrt{17})^2 - [(20 \times 20) + (15 \times 60 \div 2)]$

= $1700\pi - 400 - 450$

= $(1700\pi - 850)$ cm$^2$

The area inside the circle but outside the shaded figure is $(1700\pi - 850)$ cm$^2$ or approximately 4491 cm$^2$. 