



Problem of the Week Grade 11 and 12

An Uphill Battle Solution

Problem

Lisa-Mai Winn and Will Race are training for a competition by running all the way up and down a 700 m long ski slope. They each run up the slope at different constant speeds. Coming down the slope, each runs at double her or his uphill speed. Lisa-Mai reaches the top first, and immediately starts running back down, meeting Will 70 m from the top. When Lisa-Mai reaches the bottom, how far behind is Will?

Solution

Let a represent Lisa-Mai's uphill speed and $2a$ represent her downhill speed. Let b represent Will's uphill speed and $2b$ represent his downhill speed.

We know that $distance = speed \times time$ or $time = distance \div speed$.

When Lisa and Will meet 70 m from the top of the hill, their times are the same. At this time Will has run $700 - 70 = 630$ m uphill.

Lisa's Time Up 700 m + Lisa's Time Down 70 m = Will's Time Up 630 m

$$\frac{700}{a} + \frac{70}{2a} = \frac{630}{b}$$

Reducing the second fraction:
$$\frac{700}{a} + \frac{35}{a} = \frac{630}{b}$$

Simplifying:
$$\frac{735}{a} = \frac{630}{b}$$

By rearranging, we obtain $\frac{a}{b} = \frac{735}{630} = \frac{7}{6}$. (1)

Solution continues on the next page.





Let x represent the distance Will is behind Lisa at the end of the run.

When Lisa reaches the bottom, she has travelled 700 m up at a speed of a and 700 m down at a speed of $2a$. Will has travelled 700 m up at a speed of b and $(700 - x)$ m down at a speed of $2b$.

When Lisa reaches the bottom and Will is x m from the bottom, their times are the same.

$$\begin{aligned}\frac{700}{a} + \frac{700}{2a} &= \frac{700}{b} + \frac{700 - x}{2b} \\ \frac{1400}{2a} + \frac{700}{2a} &= \frac{1400}{2b} + \frac{700 - x}{2b} \\ \frac{2100}{2a} &= \frac{2100 - x}{2b}\end{aligned}$$

Multiplying by 2,
$$\frac{2100}{a} = \frac{2100 - x}{b}$$

It follows that $\frac{a}{b} = \frac{2100}{2100 - x}$ and $\frac{a}{b} = \frac{7}{6}$ from (1) earlier.

$$\therefore \frac{2100}{2100 - x} = \frac{7}{6}$$

Solving for x ,

$$\begin{aligned}12600 &= 14700 - 7x \\ 7x &= 2100 \\ x &= 300\end{aligned}$$

Therefore Will Race is 300 m behind Lisa-Mai Winn when she completes the run.

