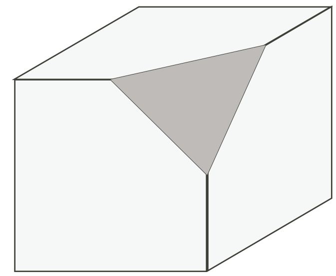
Problem of the Week
Grade 11 and 12

Cutting Corners
Solution

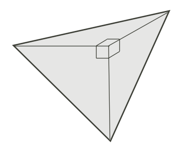
Problem

A cube with side lengths 4 cm is to be cut in the following manner: the corner of the cube is removed by making a cut through the midpoints of three adjacent sides. The diagram shows the resulting figure after the first cut. Similar cuts are made at each of the remaining corners of the cube. Determine the increase or decrease in total surface area as a result of slicing the eight corners off the original cube.

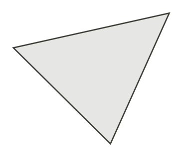


Solution

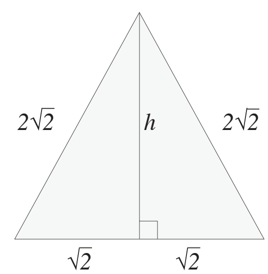
To determine the increase or decrease in surface area, we need only look at one corner, find the surface area increase or decrease there and multiply the result by eight to account for the eight corners. At each corner, the surface area of three right angled triangles is removed and replaced by the surface area of a single triangle.



Since the cut is made from the midpoints of three adjacent sides, each right triangle has base and height with length 2 cm. The resulting area of one of the right triangles is $\frac{1}{2}(2)(2) = 2\text{ cm}^2$.



The hypotenuse of the right triangle is found using Pythagoras' Theorem, $\sqrt{2^2+2^2} = \sqrt{8} = 2\sqrt{2}$. Since the length of the hypotenuse is the same in each of the three right triangles, the remaining triangle in the corner is equilateral with side lengths $2\sqrt{2}$. The triangle looks like the bottom triangle shown to the right.



The altitude of an equilateral triangle right bisects the base. Let h be the height of the triangle. Using Pythagoras' Theorem, $h^2 = (2\sqrt{2})^2 - (\sqrt{2})^2 = 8 - 2 = 6$. Therefore $h = \sqrt{6}$.

The area of the remaining triangle is $\frac{1}{2}(2\sqrt{2})(\sqrt{6}) = \sqrt{12} = 2\sqrt{3}$.

In each corner, the new surface area is the area of the equilateral triangle minus the area of the three right triangles. $\therefore$ the remaining corner area is $2\sqrt{3} - 3 \times (2) = 2\sqrt{3} - 6$. Since $2\sqrt{3} < 6$, this result is negative and the surface area is decreased in each corner. Therefore the surface area is reduced by $6 - 2\sqrt{3}$ in each corner.

Since there are eight corners, the total decrease in surface area is
$8 \times (6 - 2\sqrt{3}) = 48 - 16\sqrt{3} \doteq 20.3\text{ cm}^2$.

Therefore, as a result of cutting off each of the corners, the total surface area decreases by $(48 - 16\sqrt{3})\text{ cm}^2$.