



Problem of the Week

Grade 11 and 12

Palindrome Possibilities

Solution

Problem

A *palindrome* is a word or phrase which reads the same frontwards and backwards. “I prefer pi” is an excellent example of a palindrome phrase. Numbers which remain the same when the digits are reversed are also considered to be palindromes. For example, 1287821 and 4554 are palindromic numbers. Hannah and Habibah have a special interest in palindromic numbers. Determine, for them, the probability that a five-digit integer greater than or equal to 10 000 is a palindrome.

Solution

First we must determine how many five-digit numbers greater than or equal to 10 000 there are. The largest five-digit number is 99 999 so there are 99 999 numbers from 1 to the largest five-digit number. This includes the 9 999 numbers from 1 to 9 999 which must be excluded. So the number of five-digit numbers is $99\,999 - 9\,999 = 90\,000$.

Now we can determine how many five-digit palindromes there are. These numbers look like *abcba*. Once the first digit is selected, the last digit is known (and there is really no choice). Once the second digit is chosen, the fourth digit is known (and again there is really no choice).

There are 9 choices for the first digit. Zero cannot be used as a first digit since the number must be greater than or equal to 10 000.

For each of the 9 choices for the first digit, there are 10 choices for the second digit. Therefore, there are $9 \times 10 = 90$ choices for the first two digits.

For each of the 90 choices for the first two digits, there are 10 choices for the third digit. Therefore, there are $90 \times 10 = 900$ choices for the first three digits.

Once the first three digits are chosen there is exactly one choice for each of the fourth and fifth digits. The fourth digit must be the same as the second digit and the fifth digit must be the same as the first digit. This means that there are $900 \times 1 \times 1 = 900$ five-digit palindromes greater than or equal to 10 000.

The probability is calculated by dividing the number of five-digit palindromes greater than or equal to 10 000 by the number of five-digit numbers greater than or equal to 10 000.

$$\therefore \text{the probability that a five digit number is a palindrome} = \frac{900}{90\,000} = \frac{1}{100}.$$

