Problem of the Week
Grade 11 and 12

Answer the Question Asked
Solution

Problem

It is given that $(x+y)^2 = 16$, $(y+z)^2 = 36$, and $(x+z)^2 = 81$.
If $x+y+z \geq 1$, determine the **number** of possible values for $x+y+z$.

Solution

Since $(x+y)^2 = 16$, $x+y = \pm 4$. Since $(y+z)^2 = 36$, $y+z = \pm 6$. And since $(x+z)^2 = 81$, $x+z = \pm 9$.

Now $(x+y)+(y+z)+(x+z) = 2x+2y+2z = 2(x+y+z)$. This quantity is two times the value of the quantity we are looking for.

The following chart summarizes all possible combinations of values for $x+y$, $y+z$, and $x+z$ and the resulting values of $2x+2y+2z$ and $x+y+z$. The final column of the chart states a yes or no answer to whether the value of $x+y+z$ is ≥ 1.

$x+y$	$y+z$	$x+z$	$2x+2y+2z$	$x+y+z$	≥ 1? (yes / no)
4	6	9	19	9.5	*yes*
4	6	−9	1	0.5	*no*
4	−6	9	7	3.5	*yes*
4	−6	−9	−11	−5.5	*no*
−4	6	9	11	5.5	*yes*
−4	6	−9	−7	−3.5	*no*
−4	−6	9	−1	−0.5	*no*
−4	−6	−9	−19	−9.5	*no*

Therefore there are three possible values of $x+y+z$ such that $x+y+z \geq 1$. It should be noted that for each of the three possibilities, values for x, y, and z which produce each value can be determined.