Problem of the Week
Grade 11 and 12

Endless Possibilities?
Solution

Problem

The digits 1, 2, 3, 4, and 5 are each used once to create a five digit number $abcde$ which satisfies the following conditions: (i) the three digit number abc is odd; (ii) the three digit number bcd is divisible by 5; and (iii) the three digit number cde is divisible by 3. Determine every five digit number formed using each of the digits 1 to 5 exactly once that satisfies all three conditions.

Solution

It turns out that the possibilities are really not endless. In fact, the possibilities are quite limited.

In condition (ii), bcd is divisible by 5. For this to happen, $d = 5$ is the only possibility. $\therefore$ the number looks like $abc5e$.

Since, by condition (i), abc is odd, c must be an odd number and thus c must be 1, 3 or 5. But each digit can be used only once so 5 is eliminated. $\therefore$ the number looks like $ab15e$ or $ab35e$.

For a number to be divisible by 3, the sum of its digits must be divisible by 3. In condition (iii), cde must be divisible by 3 so the sum $c + d + e$ must be divisible by 3.

In the number $ab15e$, $cde = 15e$ and the only possible value for e such that $15e$ is divisible by 3 is $e = 3$. So $cde = 153$.

In the number $ab35e$, $cde = 35e$ and for $35e$ to be divisible by 3, e could be either 1 or 4. So $cde = 351$ or $cde = 354$.

There are then only 3 possible values for the last three digits: 153, 351 or 354.

The remaining two digits can be placed in position a or b in either order. So for each of the numbers 153, 351 and 354, there are two possible first digits giving a total of 6 possible numbers that satisfy all three conditions: 24153, 42153, 24351, 42351, 12354, and 21354.

$\therefore$ the 6 numbers that satisfy all three conditions are 24153, 42153, 24351, 42351, 12354, and 21354.