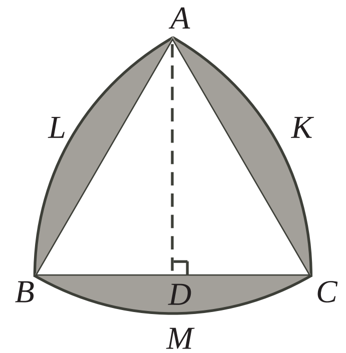
Problem of the Week
Grade 11 and 12

A Shady Space
Solution

Problem



$\triangle ABC$ is an equilateral triangle with sides of length 2 cm. ALB, BMC, and CKA are arcs of circles having centres C, A, and B, respectively. Determine the total area of the shaded regions in the diagram.

Solution

First, determine the area of equilateral $\triangle ABC$. Construct the altitude AD. Since the triangle is equilateral, all angles are $60°$. Therefore, $\triangle ABD$ is a $30°$, $60°$, $90°$ triangle with sides in ratio $1 : \sqrt{3} : 2$.

Since $AB = 2$, $BD = 1$, we get $AD = \sqrt{3}$, and

$$\text{area } \triangle ABC = (BC)(AD) \div 2 = (2)(\sqrt{3}) \div 2 = \sqrt{3} \text{ cm}^2$$

Each sector $ABMC$, $BCKA$ and $CALB$ has the same radius, 2 cm, and a $60°$ central angle. Therefore each sector has the same area, $60 \div 360$ or one-sixth the area of a circle of radius 2 cm.

$$\text{area } ABMC = \frac{1}{6}\pi r^2 = \frac{1}{6}\pi(2)^2 = \frac{2}{3}\pi \text{ cm}^2$$

The shaded part of each sector is equal to the area of the sector minus the area of the equilateral $\triangle ABC$.

$$\begin{aligned} \therefore \text{Total Shaded Area} &= 3(\text{area of any whole sector - area of the equilateral triangle}) \\ &= 3(\text{area of sector } ABMC - \text{area } \triangle ABC) \\ &= 3\left(\frac{2}{3}\pi - \sqrt{3}\right) \\ &= (2\pi - 3\sqrt{3}) \text{ cm}^2 \end{aligned}$$