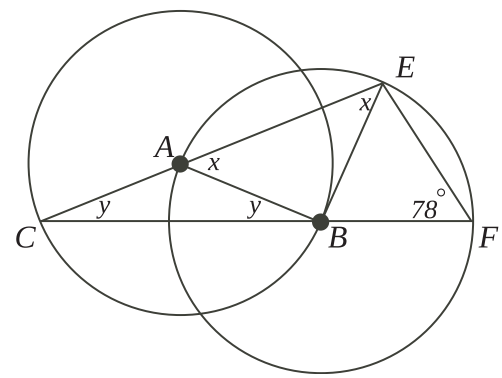
Problem of the Week
Grade 11 and 12

What's Your Angle
Solution



Problem

A is the centre of a circle which passes through B. B is the centre of a circle which passes through A. CAE and CBF are straight line segments. If $\angle F = 78^\circ$, determine the measure of $\angle C$.

Solution

Construct AB and BE. A, E and F are on the circumference of the circle with centre B. Therefore, $BA = BE = BF$.

In $\triangle BEF$, $BE = BF$. Then $\triangle BEF$ is isosceles and $\angle BEF = \angle F = 78^\circ$.

Let $x = \angle BEA$ and $y = \angle ABC$.

In $\triangle BAE$, $BA = BE$ and the triangle is isosceles. Therefore, $\angle BAE = \angle BEA = x$.

Since B and C are on the circle with centre A, $AC = AB$ and $\triangle ABC$ is isosceles. Therefore, $\angle C = \angle ABC = y$.

$\angle EAB$ is an exterior angle to $\triangle ABC$. By the exterior angle theorem for triangles, $\angle EAB = \angle C + \angle ABC$. But $\angle EAB = x$ and $\angle C + \angle ABC = y + y = 2y$. Therefore, $x = 2y$.

In $\triangle CEF$,

$$\angle C + \angle E + \angle F = 180^\circ$$
$$\angle C + \angle CEB + \angle BEF + 78^\circ = 180^\circ$$
$$y + x + 78^\circ + 78^\circ = 180^\circ$$

But $x = 2y$,

$$\therefore y + 2y + 156^\circ = 180^\circ$$
$$3y = 24^\circ$$
$$y = 8^\circ$$

Therefore, $\angle \mathbf{C} = 8^\circ$.