



Problem of the Week

Grade 11 and 12

You Make a Difference

The sequence $a_1 = 2, a_2 = 5, a_3 = 12, \dots, a_n, \dots$, where a_n is the n^{th} term in the sequence, has the property that the difference between consecutive terms forms an arithmetic sequence.

Find a formula for a_n in terms of n .

The following information *may* be helpful in the solution of the problem.

An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant. For example, 3,5,7,9 is an arithmetic sequence with four terms and constant difference 2.

The general term of an arithmetic sequence is $t_n = a + (n - 1)d$, where a is the first term, d is the constant difference and n is the number of terms.

The sum, S_n , of the first n terms of an arithmetic sequence can be found using either $S_n = \frac{n}{2}[2a + (n - 1)d]$ or $S_n = n \left(\frac{t_1 + t_n}{2} \right)$, where t_1 is the first term of the sequence and t_n is the n^{th} term of the sequence.

The following example is provided to verify the accuracy of the formulas and to illustrate their use.

For the arithmetic sequence 3,5,7,9, $a = t_1 = 3$, $d = 2$, $n = 4$ and $t_n = t_4 = 9$.

$$S_n = 3 + 5 + 7 + 9 = 24$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{4}{2}[2(3) + (3)2] = 2[12] = 24$$

$$S_n = n \left(\frac{t_1 + t_n}{2} \right) = 4 \left(\frac{3 + 9}{2} \right) = 4(6) = 24$$

