



Problem of the Week

Grade 5 and 6

How Many Combinations?

Solution

Problem

A test has 30 multiple-choice questions. A correct answer is awarded 5 points, but for every incorrect answer, you lose 2 points. An unanswered question gets 0 points. Laura received a total of 98 points. Determine the number of different ways Laura could have answered the questions in order to achieve her score.

Solution

Since Laura received a total of 98 points, then she received *at least* 98 points from her correct answers.

Since $19 \times 5 = 95$ and $20 \times 5 = 100$, then she got at least 20 questions right.

If Laura got 20 questions right, then she received $20 \times 5 = 100$ points for these questions. This means that she lost $100 - 98 = 2$ points for her wrong answers and so got 1 question wrong. Since there were 30 questions in total, then she left $30 - 20 - 1 = 9$ questions unanswered.

If Laura got 21 questions right, then she received $21 \times 5 = 105$ points for these questions. This means that she lost $105 - 98 = 7$ points for her wrong answers. But each wrong answer takes away 2 points, so it is impossible for her to lose 7 points for wrong answers, since 7 is not a multiple of 2. So Laura cannot have gotten 21 questions right.

If Laura got 22 questions right, then she received $22 \times 5 = 110$ points for these questions. This means that she lost $110 - 98 = 12$ points for her wrong answers and so got 6 questions wrong. Since there were 30 questions in total, then she left $30 - 22 - 6 = 2$ questions unanswered.

If Laura got 23 or more questions right, then she received at least $23 \times 5 = 115$ points for these questions. This means that she lost at least $115 - 98 = 17$ points for her wrong answers. Since each wrong answer is worth 2 points, then she must have gotten at least 9 questions wrong (since $9 \times 2 = 18$). But $23 + 9 = 32$, which is more than the total number of questions on the test, so this is impossible.

Therefore Laura can obtain her test score in two possible ways. The possibilities were 20 correct, 1 incorrect and 9 unanswered or 22 correct, 6 incorrect and 2 unanswered.

Refer to the next page for an extension to this problem.





Extension

John claims he scored 142. Is his claim correct? Explain your answer.

Solution to Extension

Since John scored 142, then he must have answered at least 29 questions correctly, since $28 \times 5 = 140$ and $29 \times 5 = 145$.

Since there were 30 questions on the test, then John answered either 29 or 30 questions correctly.

If he answered all 30 questions correctly, then his score would have been $30 \times 5 = 150$, not 142.

If he answered 29 questions correctly, then he got $29 \times 5 = 145$ points for these questions, and answered at most 1 question incorrectly.

In this case, he lost at most 2 points for his incorrect answer, not the 3 points that he would need to have lost for a score of 142 (since $145 - 142 = 3$).

Therefore, John cannot have obtained a score of 142 and his claim is incorrect.

