Problem of the Week
Grade 7 and 8

Some Wet Weather Math?
Solution

Problem

Raynor Schein has a bag that contains three black marbles, five gold marbles, two purple marbles, and six red marbles. During the course of cleaning his room one wet spring day, Raynor finds some white marbles and adds them to the bag. Raynor tells his friend April Showers that if she now draws a marble at random from the bag, the probability of it being black or gold is $\frac{2}{7}$. How many white marbles did Raynor Schein add to the bag?

Solution 1

At present there are $3 + 5 + 2 + 6 = 16$ marbles in the bag. Since after adding some marbles to the bag the probability of picking a black or gold marble is $\frac{2}{7}$, this implies that the new total number of marbles in the bag is a multiple of 7 and this multiple must be greater than 16. Therefore there are possibly 21, 28, 35, 42, 49, 56, $\cdots$ marbles in the bag.

There are a total of $3 + 5 = 8$ black and gold marbles in the bag. If k is the total number of marbles in the bag after adding some white marbles, then $\frac{8}{k} = \frac{2}{7}$ but $\frac{2}{7} = \frac{8}{28}$ so $\frac{8}{k} = \frac{8}{28}$ and $k = 28$ follows.

Since there were 16 marbles in the bag and there are now 28 marbles in the bag, Raynor added $28 - 16 = 12$ white marbles to the bag.

Therefore, Raynor added 12 white marbles to the bag.

Solution 2

Let w be the number of white marbles added to the bag.

There are now $3 + 5 + 2 + 6 + w = 16 + w$ marbles in the bag and $3 + 5 = 8$ black and gold marbles in the bag. We know that the number of black and gold marbles divided by the total number of marbles in the bag is $\frac{2}{7}$ so $\frac{8}{16 + w} = \frac{2}{7}$.

"Cross-multiplying" we get $(8)(7) = (2)(16 + w)$ which simplifies to $56 = 32 + 2w$ and $24 = 2w$. The result $w = 12$ follows.

Therefore, Raynor added 12 white marbles to the bag.