

Problem of the Week

Grade 7 to 8

Rectangles Abound

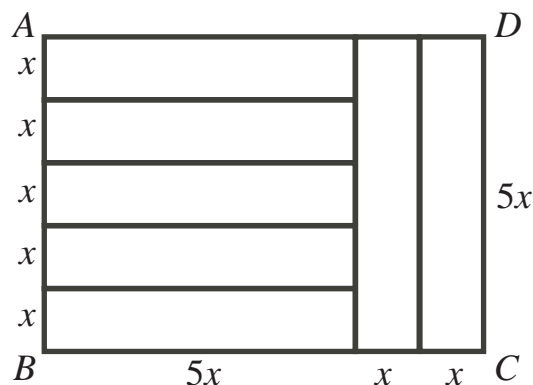
Solution

Problem

Seven identical rectangles are arranged as shown in the diagram to form a large rectangle $ABCD$. If the area of rectangle $ABCD$ is 560 cm^2 , determine the dimensions of the smaller rectangles.

Solution 1

Let x be the width of one of the smaller identical rectangles, in cm. Five of the smaller rectangles are stacked on top of each other creating AB , so $AB = x + x + x + x + x = 5x$. Since $ABCD$ is a rectangle, $AB = CD = 5x$. But CD is the length of the smaller rectangle. Therefore, the smaller rectangle is $5x$ cm by x cm.



The area of rectangle $ABCD$ is the same as 7 times the area of one of the smaller rectangles.

$$\text{Area } ABCD = 7 \times \text{Area of one smaller rectangle}$$

$$560 = 7 \times 5x \times x$$

$$560 = 35 \times x^2$$

Dividing both sides by 35, we obtain $x^2 = 16$ and $x = 4$ follows. ($x > 0$ since x is the width of the smaller rectangle.)

The width of the smaller rectangle is $x = 4$ cm and the length of the smaller rectangle is $5x = 5(4) = 20$ cm.

Therefore, the smaller rectangle is 20 cm long and 4 cm wide.

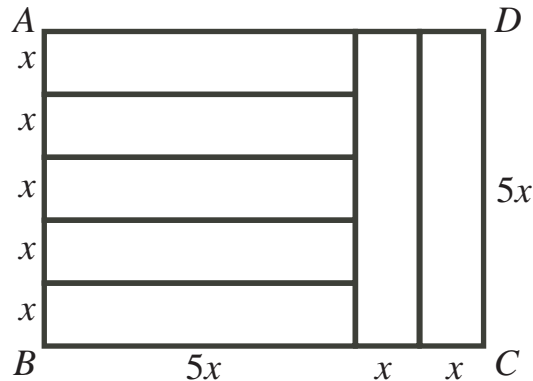




Solution 2

Let x be the width of one of the smaller identical rectangles, in cm. Five rectangles are stacked on top of each other creating AB , so

$$AB = x + x + x + x + x = 5x.$$



But $ABCD$ is a rectangle so $AB = CD$. The width of rectangle $ABCD$ is $CD = 5x$.

Now BC is made up of the length of the smaller rectangle plus two widths of the smaller rectangle. Therefore, $BC = 5x + x + x = 7x$ and rectangle $ABCD$ is $7x$ cm long and $5x$ cm wide.

To find the area of $ABCD$ we multiply the length BC by the width CD .

$$\begin{aligned}\text{Area } ABCD &= BC \times CD \\ 560 &= (7x) \times (5x) \\ 560 &= 7 \times 5 \times (x) \times (x) \\ 560 &= 35 \times x^2\end{aligned}$$

Dividing by 35, we obtain $x^2 = 16$ and $x = 4$ follows. ($x > 0$ since x is the width of the smaller rectangle.)

The width of the smaller rectangle is $x = 4$ cm and the length of the smaller rectangle is $5x = 5(4) = 20$ cm.

Therefore, the smaller rectangle is 20 cm long and 4 cm wide.

