

## Problem of the Week

### Grade 7 and 8

#### What's Your Angle I? Solution

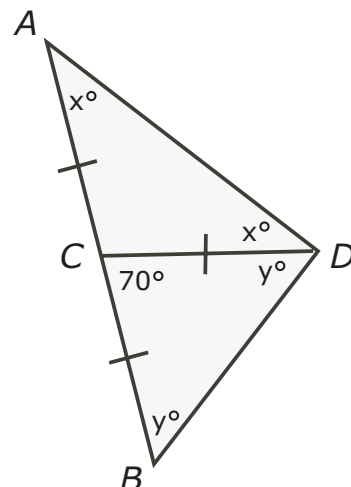
#### Problem

In  $\triangle ABD$ ,  $C$  is on  $AB$  such that  $AC = CB = CD$  and  $\angle BCD = 70^\circ$ . Determine the measure of  $\angle ADB$ .

#### Solution

Since  $ACB$  is a straight line,  $\angle ACD + \angle DCB = 180^\circ$  but  $\angle DCB = 70^\circ$  so  $\angle ACD = 110^\circ$ .

In  $\triangle ACD$ , since  $AC = CD$ ,  $\triangle ACD$  is isosceles and  $\angle CAD = \angle CDA = x^\circ$ .



The angles in a triangle sum to  $180^\circ$  so in  $\triangle ACD$

$$\begin{aligned}
 \angle CAD + \angle CDA + \angle ACD &= 180^\circ \\
 x^\circ + x^\circ + 110^\circ &= 180^\circ \\
 2x &= 70 \\
 x &= 35
 \end{aligned}$$

Similarly, in  $\triangle BCD$ , since  $CB = CD$ ,  $\triangle BCD$  is isosceles and  $\angle CBD = \angle CDB = y^\circ$ .

The angles in a triangle sum to  $180^\circ$  so in  $\triangle CBD$

$$\begin{aligned}
 \angle CBD + \angle CDB + \angle BCD &= 180^\circ \\
 y^\circ + y^\circ + 70^\circ &= 180^\circ \\
 2y &= 110 \\
 y &= 55
 \end{aligned}$$

Then  $\angle ADB = \angle CDA + \angle CDB = x^\circ + y^\circ = 35^\circ + 55^\circ = 90^\circ$ .

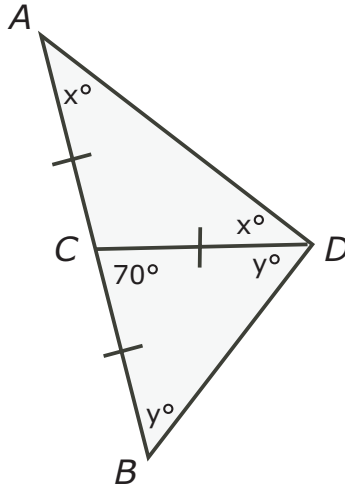
$\therefore$  the measure of  $\angle ADB$  is  $90^\circ$ .

See the next page for an interesting idea.





It turns out that it is not necessary to find the angles in the problem.



Here is a second solution to the problem.

In  $\triangle CAD$ , since  $CA = CD$ ,  $\triangle CAD$  is isosceles and  $\angle CAD = \angle CDA = x^\circ$ .

In  $\triangle CBD$ , since  $CB = CD$ ,  $\triangle CBD$  is isosceles and  $\angle CBD = \angle CDB = y^\circ$ .

The angles in a triangle sum to  $180^\circ$  so in  $\triangle ABD$

$$\begin{aligned}\angle BAD + \angle ADB + \angle ABD &= 180^\circ \\ x^\circ + (x^\circ + y^\circ) + y^\circ &= 180^\circ \\ 2x^\circ + 2y^\circ &= 180^\circ \\ x^\circ + y^\circ &= 90^\circ\end{aligned}$$

But  $\angle ADB = \angle ADC + \angle CDB = x^\circ + y^\circ = 90^\circ$ .

$\therefore$  the measure of  $\angle ADB$  is  $90^\circ$ .

