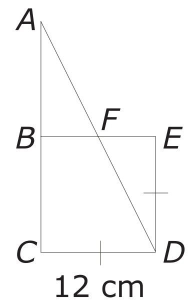
Problem of the Week
Grade 7 and 8

All Things Being Equal
Solution

Problem



Square $BCDE$ and $\triangle ACD$ have equal areas. Square $BCDE$ has sides of length 12 cm. AD intersects BE at F. Determine the area of quadrilateral $BCDF$.

Solution

The area of square $BCDE = 12 \times 12 = 144$ cm^2. The area of $\triangle ACD$ equals the area of square $BCDE$. Therefore area $\triangle ACD = 144$ cm^2.

The area of a triangle is calculated using the formula *base* $\times$ *height* $\div 2$. It follows that:

$$\begin{aligned} \text{Area } \triangle ACD &= (CD) \times (AC) \div 2 \\ 144 &= 12 \times AC \div 2 \\ 144 &= 6 \times AC \\ 24 \text{ cm} &= AC \end{aligned}$$

But $AC = AB + BC$ so $24 = AB + 12$ and it follows that $AB = 12$ cm.

$$\begin{aligned} \text{Area } \triangle ACD &= \text{Area Square } BCDE \\ \text{Area } \triangle ABF + \text{Area Quad. } BCDF &= \text{Area } \triangle DEF + \text{Area Quad. } BCDF \end{aligned}$$

Since the area of quadrilateral $BCDF$ is common to both sides of the equation, it follows that the area of $\triangle ABF$ equals the area of $\triangle DEF$.

$$\begin{aligned} \text{Area } \triangle ABF &= \text{Area } \triangle DEF \\ (AB) \times (BF) \div 2 &= (DE) \times (EF) \div 2 \\ 12 \times BF \div 2 &= 12 \times FE \div 2 \\ 6 \times BF &= 6 \times FE \\ \therefore BF &= FE \end{aligned}$$

But $BF + FE = BE = CD = 12$ so $BF = FE = 6$ cm. Then the area of $\triangle DEF = DE \times FE \div 2 = 12 \times 6 \div 2 = 36$ cm^2.

$$\begin{aligned} \text{The area of quadrilateral } BCDF &= \text{ area of square } BCDE - \text{ area } \triangle DEF \\ &= 144 - 36 \\ &= 108 \text{ cm}^2 \end{aligned}$$

Therefore the area of quadrilateral $BCDF$ is 108 cm^2.