



Problem of the Week

Grade 7 and 8

A Crafty Little Problem

Solution

Problem

A knot is tied near one end of a piece of string and beads are strung to form a necklace. The beads are placed on the string in the following sequence: 1 red, 1 green, 1 blue, 2 red, 2 green, 2 blue, 3 red, 3 green, 3 blue, with the number of each colour increasing by one every time a new group of beads is placed. How many of the first 160 beads are blue?

Solution

An equal number of red, green and blue beads occur after

$$\begin{aligned}3(1) &= 3 \text{ beads are placed,} \\3(1) + 3(2) &= 3 + 6 = 9 \text{ beads are placed,} \\3(1) + 3(2) + 3(3) &= 3 + 6 + 9 = 18 \text{ beads are placed, and so on.}\end{aligned}$$

The greatest total that can be placed with equal numbers of red, green and blue beads is $3(1) + 3(2) + 3(3) + \dots + 3(9) = 3 + 6 + 9 + \dots + 27 = 135$. At this point there are $135 \div 3 = 45$ beads of each colour. We are at a point where we have strung 9 red beads, 9 green beads and 9 blue beads in the last three groups.

The next groups will have 10 in them if there are enough beads. There are $160 - 135 = 25$ beads left to place. We are able to place 10 red beads leaving 15 beads left to place. At this point there are 55 red beads. We can place 10 green beads leaving 5 beads left to place. At this point there are 55 green beads. The five remaining beads must be blue making the total number of blue beads 50.

\therefore there are 50 blue beads on the necklace.

