



Problem of the Week Grade 9 and 10

The Case of the Missing Digits Solution

Problem

A student discovered the question shown below on a scrap piece of paper. In both the top and bottom numbers, the middle digit was unreadable. However, there was a note on the page beside the question which read: “the sum is divisible by 3.” If the missing middle digit in the top number is A and the missing middle digit in the bottom number is B , determine all possible values for A and B .

$$\begin{array}{r} 2\ A\ 4 \\ +\ 3\ 2\ 9 \\ \hline 5\ B\ 3 \end{array}$$

Solution 1

First we can find the possible values of B . For a number to be divisible by 3, the sum of its digits must be divisible by 3. Only 513, 543 and 573 are divisible by three so the only possible values for B are 1, 4, or 7.

We know that $2A4 + 329 = 5B3$ so $2A4 = 5B3 - 329$.

We can try each of the possible values for B in the equation $2A4 = 5B3 - 329$ to find values of A that make the equation true.

1. If $B=1$, then $513 - 329 = 184$, which cannot equal $2A4$. So when $B = 1$ there is no A to satisfy the problem.
2. If $B=4$, then $543 - 329 = 214$, which does equal $2A4$ when $A = 1$. So for $A = 1$ and $B = 4$ there is a valid solution.
3. If $B=7$, then $573 - 329 = 244$, which does equal $2A4$ when $A = 4$. So for $A = 4$ and $B = 7$ there is a valid solution.

\therefore when $A = 1$ and $B = 4$ or $A = 4$ and $B = 7$, the given problem has a valid solution.





Solution 2

$$\begin{array}{r} 2 \ A \ 4 \\ + 3 \ 2 \ 9 \\ \hline 5 \ B \ 3 \end{array}$$

When the digits in the unit's column are added together, there is one carried to the ten's column. When the digits in the hundred's column are added together we get $2 + 3 = 5$ so there is no carry from the ten's column. Therefore, when the ten's column is added we get $1 + A + 2 = B$ or $A + 3 = B$.

We can now look at all possible values for A that produce a single digit value for B in the number $5B3$. We can then determine whether or not $5B3$ is divisible by 3.

The following table summarizes the results.

A	$B = A + 3$	$5B3$	Divisible by 3 (<i>yes/no</i>)?
0	3	533	<i>no</i>
1	4	543	<i>yes</i>
2	5	553	<i>no</i>
3	6	563	<i>no</i>
4	7	573	<i>yes</i>
5	8	583	<i>no</i>
6	9	593	<i>no</i>

\therefore when $A = 1$ and $B = 4$ or $A = 4$ and $B = 7$, the given problem has a valid solution.

