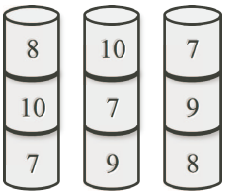
Problem of the Week
Grade 9 and 10

Step Right Up
Solution

Problem



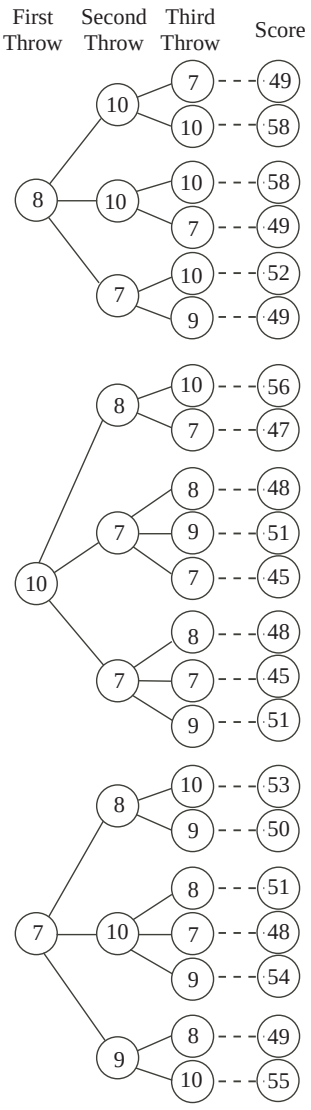
In a certain carnival game nine numbered paint cans are stacked as shown. The rules of the game are as follows:

1. Each player gets exactly three turns, one throw per turn.
2. On each throw you are allowed to knock down exactly one can. If your throw hits any can but the top can you will knock down more than one can and lose. If you completely miss the cans on a turn it is not possible to score enough points to win and therefore you lose.
3. Your score on the first throw is the number on the can that you knocked down. Your score on the second throw is two times the number on the can that you knocked down. Your score on the final throw is three times the number on the can that you knocked down.
4. You are a winner if the sum of your scores from your three turns is exactly 50 points. If the sum is under 50 points, you lose. If the sum is over 50 points, you lose.

Describe precisely the different ways you can win the game. Good luck - everyone's a winner!

Solution

On your first throw, you knock down either an $8, 10$ or 7. If you get an 8, you can get a $10, 10$ or 7 on your second throw. If you get an 8 on your first throw, and a 10 on your second throw, you can get either a 7 or 10 on your third throw. The flow diagram to the right illustrates all of the possibilities for your three throws.



Once you have your possible outcomes, you can compute a score by adding the value of your first throw to twice the value of your second throw and three times the value of your third throw. For example, if your three throws are 8, 7 and 9, your score is calculated $8 + 2(7) + 3(9) = 8 + 14 + 27 = 49$. You would lose with this score. All possible scores are calculated in the last column of the flow diagram.

It turns out that there is only one possible combination of throws which produces a winning score. You can win if your first throw hits the 7 on the top of the third column of cans followed by a second throw hitting the 8 on the first column of cans and finally a third throw hitting the 9 on the third column of cans. Your winning score is calculated $7 + 2(8) + 3(9) = 7 + 16 + 27 = 50$.

The claim that "everyone's a winner" does not really apply to this carnival game.