Problem of the Week
Grade 9 and 10

It's a Trap!
Solution

Problem

A four-sided figure with one pair of parallel sides is referred to as a trapezoid. Trapezoid $ABCD$ has three equal sides $AB = AD = DC$. The base BC is 2 cm less than the sum of the lengths of the other three sides. The distance between the parallel sides is 5 cm. Determine the area of trapezoid $ABCD$.

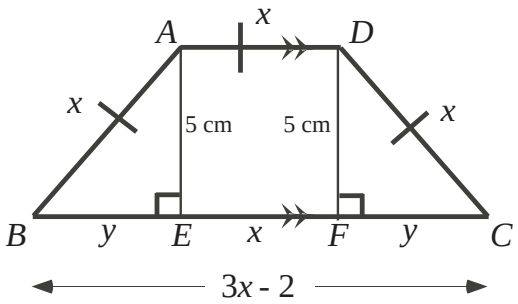
Solution

Let x represent the length of AB. Then $AB = AD = DC = x$. Since the base BC is two less than the sum of the three equal sides, $BC = 3x - 2$.

Construct altitudes from A and D meeting BC at E and F, respectively. Then $AE = DF = 5$, the distance between the two parallel sides.

Let y represent the length of BE. We can show that $BE = FC$ using the Pythagorean Theorem as follows: $BE^2 = AB^2 - AE^2 = x^2 - 5^2 = x^2 - 25$ and $FC^2 = DC^2 - DF^2 = x^2 - 5^2 = x^2 - 25$. Then $FC^2 = BE^2 = x^2 - 25$ and $FC = BE = y$.

Since $\angle AEF = \angle DFE = 90°$ and AD is parallel to EF, it follows that $\angle DAE = \angle ADF = 90°$ and $AEFD$ is a rectangle so $EF = AD = x$. The following diagram contains all of the given and found information.



We can now determine a relationship between x and y.

$$\begin{aligned} BC &= BE + EF + FC \\ 3x - 2 &= y + x + y \\ 3x - 2 &= 2y + x \\ 2x - 2 &= 2y \\ x - 1 &= y \end{aligned}$$

In right $\triangle ABE$, $AB^2 = AE^2 + BE^2$ or $x^2 = y^2 + 5^2$ or $x^2 = (x-1)^2 + 25$ or $x^2 = x^2 - 2x + 1 + 25$ or $2x = 26$ or $x = 13$. Since $x = 13$, $3x - 2 = 3(13) - 2 = 37$. Then $AD = x = 13$ and $BC = 3x - 2 = 37$.

$\therefore$ The area of trapezoid $ABCD = AE \times (AD + BC) \div 2 = 5 \times (13 + 37) \div 2 = 125$ cm^2.