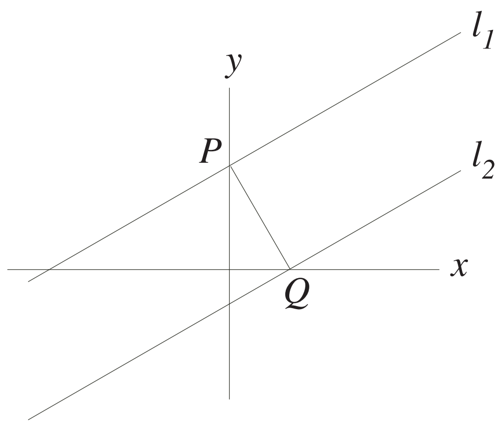
Problem of the Week
Grade 9 and 10

Slippery Slopes
Solution

Problem

Line l_1 has equation $y = mx + k$. Line l_1 crosses the y-axis at point P and l_2 crosses the x-axis at point Q. PQ is perpendicular to both line l_1 and line l_2. Determine the y-intercept of l_2 in terms of m and k.



Solution

Let the y-intercept of l_2 be represented by b.

l_1 has equation $y = mx + k$ so we know the slope of l_1 is m and the y-intercept is k. Therefore P, the y-intercept of l_1, is the point $(0,k)$.

Since $PQ \perp l_1$ and l_2, it follows that $l_1 \parallel l_2$. Also the slope of PQ is the negative reciprocal of the slope of l_1. Therefore, slope$(PQ) = \frac{-1}{m}$. Since k is the y-intercept of PQ and the slope of PQ is $\frac{-1}{m}$, the equation of the line through PQ is $y = \frac{-1}{m}x + k$.

The x-intercepts of PQ and l_2 are the same since both lines intersect at Q on the x-axis. To find this x-intercept set $y = 0$ in $y = \frac{-1}{m}x + k$. Then $0 = \frac{-1}{m}x + k$ and $\frac{1}{m}x = k$. The result $x = mk$ follows. The x-intercept of PQ and l_2 is mk and the coordinates of Q are $(mk, 0)$.

We can now find y-intercept of l_2 since $Q(mk, 0)$ is on l_2 and the slope of l_2 is m. Substituting into the slope-intercept form of the line, $y = mx + b$, we obtain $0 = (m)(mk) + b$ which simplifies to $b = -m^2k$.

Therefore the y-intercept of $\mathbf{l_2}$ is $\mathbf{-m^2k}$.

To the student who solved the problem for l_1 with equation $y = 4x + 3$, you should have obtained the answer -48 for the y-intercept of l_2.