Problem of the Week
Grade 9 and 10

Looking for Leftovers?
Solution

Problem

3^5 means $3 \times 3 \times 3 \times 3 \times 3$ and equals 243 when expressed as an integer. When the number 3^{2011} is divided by 5, what is the remainder?

Solution

We looked at a similar problem during the second half of the last school year that asked for the pattern of the final digits. It turns out that we can use the same idea to find the last digit of 3^{2011} and then apply the divisibility rule for division by 5.

When a number ends in 0 or 5, the remainder is 0 when divided by 5. If a number ends in 1 or 6, the remainder is 1 when divided by 5. For example, $56 \div 5 = 11$ R1. If a number ends in 2 or 7, the remainder is 2 when divided by 5. If a number ends in 3 or 8, the remainder is 3 when divided by 5. If a number ends in 4 or 9, the remainder is 4 when divided by 5.

So let's examine the pattern of the last digits on the first eight powers of 3.

Exponent	Power	Value	Final Digit
1	3^1	3	3
2	3^2	9	9
3	3^3	27	7
4	3^4	81	1
5	3^5	243	3
6	3^6	729	9
7	3^7	2187	7
8	3^8	6561	1

It would appear that there is a pattern in the final digits that repeats every four numbers. The pattern would continue: the final digits of 3^9, 3^{10}, 3^{11}, 3^{12} would be 3, 9, 7, 1, respectively. So we need to determine how many complete groups of four are in 2011. $2011 \div 4 = 502$ R3. There are 502 complete repetitions of the pattern of final digits and 3 numbers into the next pattern. This means that the final digit of 3^{2011} is 7, the third number in the pattern of the final digits. Since the final digit is 7, the remainder when 3^{2011} is divided by 5 is 2.

$\therefore$ when 3^{2011} is divided by 5, the remainder is 2.