Math in the Real World: Digital Imaging

CEMC

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We will start by discussing how these images are stored on a computer and then we will move on to how this data is transformed for different purposes.

The Binary Number System

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Let's take for example the number 473. Here's how we would break this number down in both systems:

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As you can see, binary gives us a string of 1's and 0's.
Try converting 255 to binary! Make sure you start by finding the largest power of 2 that is less than 255.

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Image source: http://www.whydomath.org/node/wavlets/images/grayrange.gif

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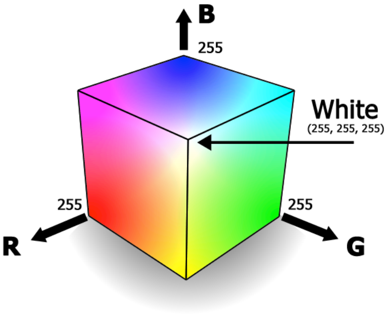

Image source: https://www.medialooks.com/mformats/docs/images/CK_color_cube.png

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Try it out:

1. What is the binary coordinate representation of Waterloo Gold?
2. What would black (on the RGB scale) be in decimal?
3. What would black (on the RGB scale) be in binary?

Storing Information: Pixels and Bits

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Take a look at two examples of PPI below:

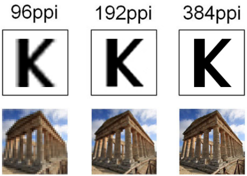

Image source: http://s01.shiftdelete.net/img/content/16-10/04/ppi_karsilastirmasi.jpg

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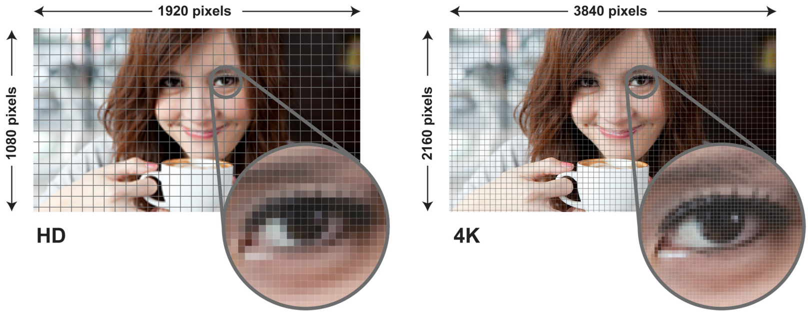

Image source: http://graphics.secondipity.com/gr/images/nw/2013X900A1.jpg

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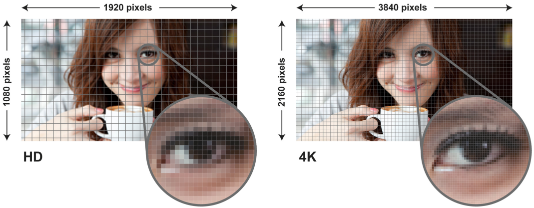
Eight bits make up one *byte*, and $2^{10} = 1024$ bytes make up one kilobyte (KB).

Furthermore, $(2^{10})^2 = 1024^2 = 1048576$ bytes make up one megabyte (MB). The megabyte, along with the gigabyte (GB) are probably the most familiar storage units.

Storing Information: Pixels and Bits



Try it out:

1. How many bits are in one black and white (greyscale) pixel?
2. How many bits are in one colour (RGB) pixel?
3. How many bits are in the HD *colour* image pictured above?
4. How many bits are in the 4K *colour* image pictured above?
5. What is the file size of the HD image in MB?
6. What is the file size of the 4K image in MB?
7. What percentage of the size of the HD file is the size of the 4K file?
8. How many bytes do you think there are in 1 GB?

A Component of Compression: Huffman Coding

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One simplistic method of data compression that is occasionally utilized: Huffman Coding.

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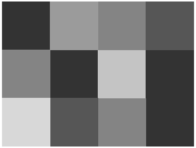
Examine the section of pixels captured below, along with its numeric greyscale values:



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132	51	196	51
216	86	132	51

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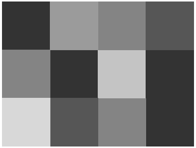

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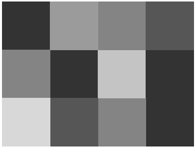

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Value	Frequency	Relative Frequency
51	4	$4/12 = 0.33$
132	3	$3/12 = 0.25$
86	2	$2/12 = 0.17$
155	1	$1/12 = 0.08$
196	1	$1/12 = 0.08$
216	1	$1/12 = 0.08$

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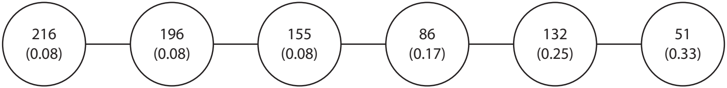
Next, we sort the values by relative frequency (smallest to largest):



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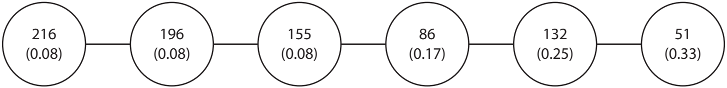
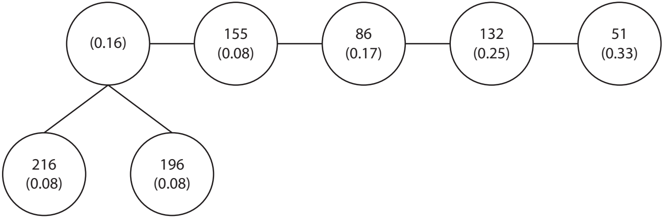


Now, add the relative frequences of the two leftmost nodes and create a new node with two children:



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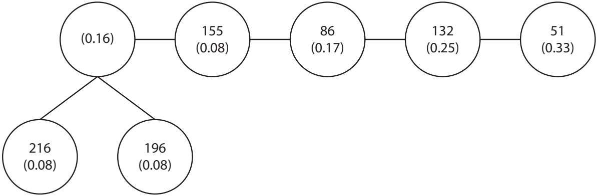
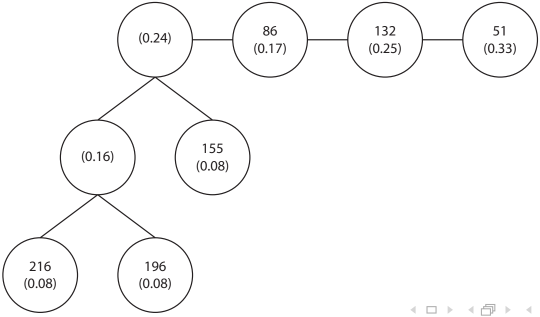


Repeat the previous step:



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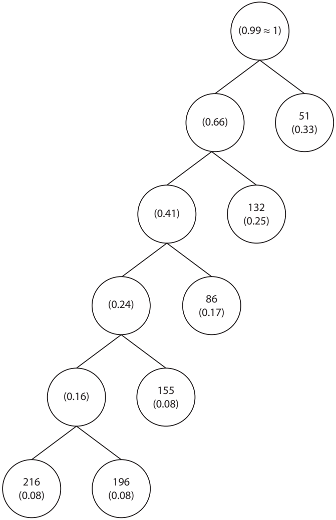
Keep repeating until there is a single node of frequency 1 at the top:



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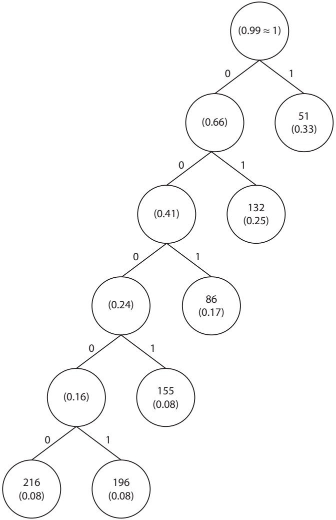
Lastly, label the branches connecting nodes. Left branches get a 0, right branches get a 1.



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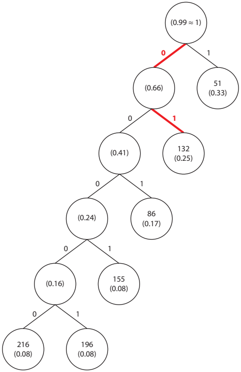
Originally, the value of 132 was 10000100. From our Huffman Coding, the new value of 132 is 01 (see the path highlighted in red below). We have saved 6 bits for every occurence of 132.



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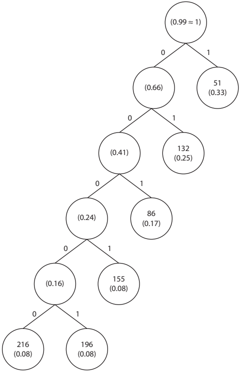

Try it out:

1. How many bits were originally used to store the section of greyscale pixels?
2. Create a table listing the original pixel values (in decimal) and their new Huffman values.
3. How many bits are used to store the Huffman coded section of pixels?
4. What percentage of the size of the original file is the size of the Huffman file?
5. What made Huffman Coding so effective with this specific section of pixels? In what situation would Huffman Coding not be as effective?
6. Create the Huffman tree for the frequency table below and list the new Huffman values.

Value	Frequency	Relative Frequency
115	5	$5/10 = 0.5$
97	3	$3/10 = 0.3$
102	1	$1/10 = 0.1$
114	1	$1/10 = 0.1$

Transforming Images with Functions

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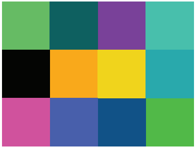
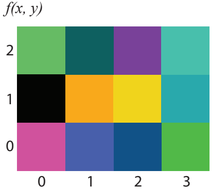
Once again, let's examine a section of pixels:



Transforming Images with Functions

Now, we're going to label the pixels with coordinates as follows:



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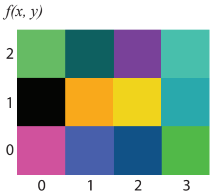

We can use function notation to represent our grid of pixels. The function $f(x, y)$ calls on the pixel located at (x, y) and gives us the RGB colour code for that pixel.

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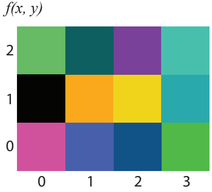

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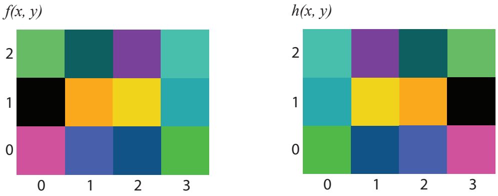
For example, $f(0, 1) = (0, 0, 0)$ which we know is black. We can use this notation to manipulate the pixels.

Transforming Images with Functions

Let's flip/mirror the section of pixels horizontally. We will call the function that represents our new section of pixels $h(x, y)$.



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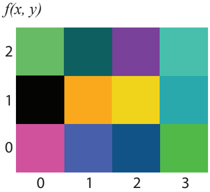

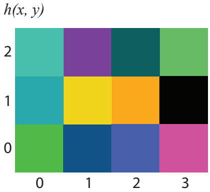

Choosing a few points as examples, we see that $h(0, 0) = f(3, 0)$, $h(1, 2) = f(2, 2)$, and $h(3, 2) = f(0, 2)$.

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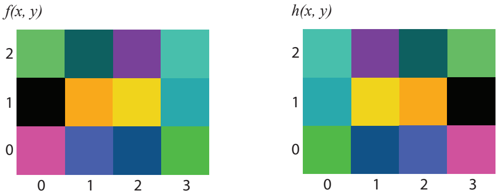

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From these sample points, we can see that the horizontally flipped image is $h(x, y) = f(3 - x, y)$.

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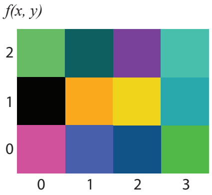

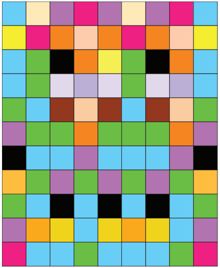

Try it out:

1. What function $v(x, y)$ flips/mirrors the original section of pixels $f(x, y)$ vertically?
2. What function $r(x, y)$ rotates $f(x, y)$ 180°? (*Hint: Don't think of it as a rotation.*)
3. The 11×9 grid of pixels $a(x, y)$ is shown to the left. Draw $b(x, y) = a(2x, 2y)$ with the restrictions $x \leq 4, y \leq 5$. (*Hint 1: $b(x, y)$ should be a 6×5 grid. Hint 2: You can immediately eliminate the 69 squares that don't show up in $b(x, y)$ if you think about what is happening with the function.*)

Thank You!

Visit **cemc.uwaterloo.ca** for great mathematics resources!