Canadian
Mathematics
Competition

An activity of The Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario

2002 Solutions
Pascal Contest (Grade 9)

for

The CENTRE for EDUCATION in MATHEMATICS and COMPUTING
Awards

1. By order of operations,
$$\frac{15+9-6}{3\times 2}=\frac{18}{6}=3$$
ANSWER: (C)

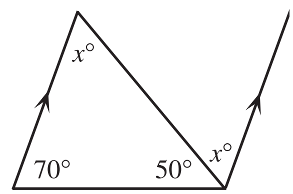
2. 50% of 2002 is $\frac{1}{2}$ of 2002, which is 1001. ANSWER: (E)

3. Since $10=x+2$, then $x=8$. Since $6=y-1$, then $y=7$. Thus, $x+y=15$.
ANSWER: (B)

4. Evaluating,
$$\left(3^2-3\right)^2=(9-3)^2=36.$$
ANSWER: (A)

5. Since Sofia goes up 7 floors, then down 6 floors, and then finally up 5 floors, the net result is that she has gone up $7-6+5=6$ floors. Since she finishes on the 20th floor, she must have started on floor number 14. ANSWER: (A)

6. Because of alternate angles, the third angle in the triangle is x°. The sum of angles in a triangle is 180°, thus



$$x^\circ+70^\circ+50^\circ=180^\circ$$
$$x^\circ=60^\circ$$
$$x=60$$
ANSWER: (B)

7. Since $n=\frac{5}{6}(240)$, then $\frac{2}{5}n=\frac{2}{5}\left(\frac{5}{6}\right)(240)=\frac{1}{3}(240)=80$. ANSWER: (B)

8. Evaluating, using the rules for negative exponents,
$$1-\left(5^{-2}\right)=1-\frac{1}{5^2}=1-\frac{1}{25}=\frac{24}{25}.$$
ANSWER: (A)

9. We can determine the area of the shaded region by guessing the side lengths of the various rectangles and then cross-checking our results. Let us suppose that the top left rectangle has a width of 2 and a height of 3. Then the top right rectangle has a width of 5, since its height is also 3. Thus, we can conclude that the height of the bottom right rectangle is 5. This tells us that the shaded rectangle is 2 by 5, or has an area of 10. This problem can also be solved with a more algebraic approach, but this is the most straightforward way.
ANSWER: (E)

10. For 1 square, 4 toothpicks are needed.
For 2 squares, 7 toothpicks are needed.

For 3 squares, 10 toothpicks are needed.
So when each additional square (after the first) is added in the pattern, 3 more toothpicks are added. Thus, to form 10 squares, we need to add 9 groups of 3 toothpicks to the original 4, so we need $4+9(3)=31$ toothpicks in total. ANSWER: (C)

11. Since $ABCD$ is a square, then $AB=BC$, or

$$x+16=3x$$
$$16=2x$$
$$x=8$$

Thus the side length of the square is $x+16=3x=24$, and the perimeter is $4(24)=96$.

ANSWER: (C)

12. Let the first number in the list be a. Then the second and third numbers are $2a$ and $4a$, respectively. So $2a+4a=24$, or $a=4$. Now the fourth, fifth and sixth numbers will be $8a$, $16a$ and $32a$, respectively. So the sixth number is $32(4)=128$.

ANSWER: (E)

13. Side AC of ΔABC is parallel to the y-axis and so is perpendicular to the x-axis, and crosses the x-axis at $(1,0)$. So we can think of AC as the base of the triangle (length 6) and the part of the x-axis inside ΔABC as its height (length 3). So the area of ΔABC is $\frac{1}{2}(6)(3)=9$.

ANSWER: (E)

14. To calculate the overall class average, we can assume that each of the 25 students who averaged 75% each actually got a mark of 75%. Similarly, we may assume that the other 5 students each actually got a mark of 40%. So the overall average is

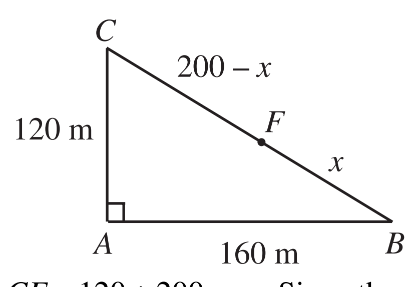
$$\frac{25(75)+5(40)}{30}=\frac{2075}{30}=69.2$$

which is closest to 69 of all of the possible choices. ANSWER: (B)

15. By Pythagoras,

$$BC^2=120^2+160^2$$
$$BC^2=40000$$
$$BC=200$$

Let $FB=x$. Then $FC=200-x$.



So Jack jogs $AB+BF=160+x$, and Jill jogs $AC+CF=120+200-x$. Since they jog the same distance,

$$160+x=120+200-x$$
$$2x=160$$
$$x=80$$

Therefore, it is 80 metres from B to F. ANSWER: (D)

16. First, we see that both 5^3 and 7^{52} are odd integers, so their product is odd. Also, 5^3 is a multiple of 5, so $\left(5^3\right)\left(7^{52}\right)$ is a multiple of 5, and thus is an odd multiple of 5. Since all odd multiples of 5 end with the digit 5, then the units digit of $\left(5^3\right)\left(7^{52}\right)$ is 5.

ANSWER: (A)

17. We know that $1000 = 10 \times 10 \times 10 = 2 \times 5 \times 2 \times 5 \times 2 \times 5$. To write 1000 as the product of two integers neither of which contains a 0, we must ensure that neither has a factor of 10. This tells us that we want to separate the factors of 2 and the factors of 5, ie.
$1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5 = 8 \times 125$, and $8 + 125 = 133$. ANSWER: (C)

18. Since Akira and Jamie together weigh 101 kg, and Akira and Rabia together weigh 91 kg, then Jamie weighs 10 kg more than Rabia. So let Rabia's weight be x kg. Then Jamie's weight is $(10 + x)$ kg. So from the third piece of information given,

$$x + x + 10 = 88$$
$$2x = 78$$
$$x = 39$$

Since Rabia weighs 39 kg, then Akira weighs 52 kg, since their combined weight is 91 kg.

ANSWER: (D)

19. If we divide 2002 by 7, we see that $2002 = 7(286)$. Since there are 7 natural numbers in each row, and the last entry in each row is the multiple of 7 corresponding to the row number, then 2002 must lie in the 7th column of the 286th row. So $m = 7$, $n = 286$, and $m + n = 293$. ANSWER: (D)

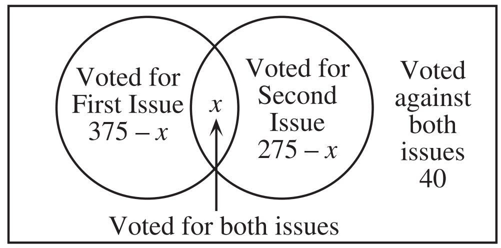
20. For $\sqrt{25 - x^2}$ to be defined, $25 - x^2 \geq 0$, or $x^2 \leq 25$, or $-5 \leq x \leq 5$. So we make a table of x and $25 - x^2$ and check to see when $25 - x^2$ is a perfect square, ie. $\sqrt{25 - x^2}$ is an integer.

x	$25 - x^2$	Perfect Square?
0	25	Yes
± 1	24	No
± 2	21	No
± 3	16	Yes
± 4	9	Yes
± 5	0	Yes

Therefore, there are 7 integer values for x (namely $0, \pm 3, \pm 4, \pm 5$) that make $\sqrt{25 - x^2}$ an integer. ANSWER: (A)

21. The original rectangular block measures 5 cm by 6 cm by 4 cm, and so has volume $5 \times 6 \times 4 = 120 \text{ cm}^3$. This tells us that it is made up of 120 of the small cubes. The largest cube that can be formed by removing cubes is a 4 cm by 4 cm by 4 cm cube, since each side length can be no more longer than the side lengths of the original block. This new cube is made up of $4 \times 4 \times 4 = 64$ of the 1 cm^3 cubes, and so 56 of the original 1 cm^3 cubes have been removed. ANSWER: (E)

22. Let x be the number of students who voted in favour of both issues. We construct a Venn diagram of the results of the vote:



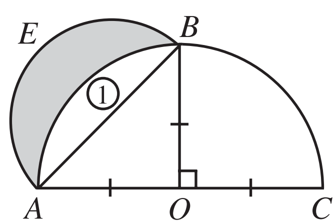
Since the total number of students is 500, then

$$375 - x + x + 275 - x + 40 = 500$$
$$690 - x = 500$$
$$x = 190$$

So 190 students voted in favour of both. ANSWER: (C)

23. We must find all ordered pairs (a,b) of integers which satisfy $a^b = 64 = 2^6$. We can first say that we know that b must be positive, since a^b is bigger than 1. However, it is possible that a is negative.
If we want both a and b to be positive, the only ways to write 64 are $64 = 64^1 = 8^2 = 4^3 = 2^6$ (since a must be a power of 2). If b is even, then it possible for a to be negative, ie. $64 = (-8)^2 = (-2)^6$. So there are 6 ordered pairs (a,b) that satisfy the equation.
ANSWER: (D)

24. The easiest way to calculate the area of the shaded region is to take the area of the semi-circular region AEB and subtract the area of the region labelled ①. But the area of this region ① is the area of the quarter circle ABO minus the area of the triangle ABO. We now calculate these areas.



Since $AO = BO = 1$, then the area of ΔABO is $\frac{1}{2}(1)(1) = \frac{1}{2}$.
Since $AO = BO = 1$, then radius of the quarter circle ABO is 1, and so the area of the quarter circle is $\frac{1}{4}\pi(1)^2 = \frac{\pi}{4}$.
Lastly, since $AO = BO = 1$, then $AB = \sqrt{2}$, and so the radius of the semi-circle AEB is $\frac{\sqrt{2}}{2}$, which means that the area of the semi-circle AEB is $\frac{1}{2}\pi\left(\frac{\sqrt{2}}{2}\right)^2 = \frac{\pi}{4}$.
Thus, the area of the shaded region is
(Area of semicircle AEB) – (Area of region ①)

$= (\text{Area of semicircle } AEB) - [(\text{Area of quarter circle } AOB) - (\text{Area of } \Delta AOB)]$
$= \frac{\pi}{4} - \left[\frac{\pi}{4} - \frac{1}{2}\right]$
$= \frac{1}{2}$

ANSWER: (B)

25. First, we calculate the volumes of the two cylindrical containers:

$V_{\text{large}} = \pi(6)^2(20) = 720\pi \text{ cm}^3$

$V_{\text{small}} = \pi(5)^2(18) = 450\pi \text{ cm}^3$

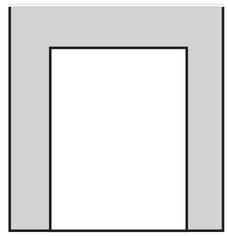


Figure 3

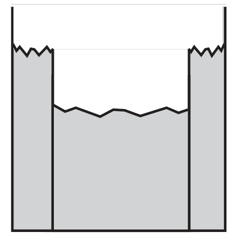


Figure 4

The volume of water initially contained in the large cylinder is

$V_{\text{water, initial}} = \pi(6)^2(17) = 612\pi \text{ cm}^3$

The easiest way to determine the final depth of water in the small cylinder is as follows. Imagine putting a lid on the smaller container and lowering it all the way to the bottom of the larger container, as shown in Figure 3. So there will be water beside and above the smaller container. Note that the larger container will be filled to the brim (since the combined volume of the small container and the initial water is greater than the volume of the large container) and some water will have spilled out of the larger container.

Now if the lid on the small container is removed, all of the water in the large container above the level of the brim of the small container will spill into the small container, as shown in Figure 4. This water occupies a cylindrical region of radius 6 cm and height 2 cm, and so has a volume of $\pi(6)^2(2) = 72\pi \text{ cm}^3$. This is the volume of water that is finally in the small container. Since the radius of the small container is 5 cm, then the depth of water is

$$\text{Depth} = \frac{72\pi \text{ cm}^3}{\pi(5 \text{ cm})^2} = \frac{72}{25} \text{ cm} = 2.88 \text{ cm}$$

ANSWER: (D)