Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

# 2003 Solutions

# Gauss Contest

*(Grades 7 and 8)*

| | |
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## Part A

1. Multiplying gives $3.26 \times 1.5 = 4.89$. Answer: (B)

2. Calculating in the parentheses first,
$$(9-2)-(4-1)=7-3=4.$$
Answer: (C)

3. Adding gives,
$$30+80\,000+700+60=30+80\,760=80\,790$$
Answer: (D)

4. $\dfrac{1+2+3}{4+5+6}=\dfrac{6}{15}=\dfrac{2}{5}$ Answer: (C)

5. In the survey, a total of 90 people were surveyed.
According to the graph, 25 people chose a cat, 10 people chose a fish, 15 people chose a bird, and 5 people chose "other", accounting for $25+10+15+5=55$ people.
This leaves $90-55=35$ people who have chosen a dog. Answer: (E)

6. If Travis uses 4 mL of gel every day and a tube of gel contains 128 mL of gel, then it will take him $\dfrac{128}{4}=32$ days to empty the tube. Answer: (A)

7. *Solution 1*
On the left hand side of the equation, if the 3's are cancelled, we would have
$$\frac{3\times 6\times 9}{3}=6\times 9=\frac{\square}{2}$$
Therefore, the expression in the box should be $2\times 6\times 9$, since we can write $6\times 9=\dfrac{2\times 6\times 9}{2}$.

*Solution 2*
Evaluating the left side of the expression, we obtain $\dfrac{3\times 6\times 9}{3}=54$.
Therefore, we must place an expression equal to $54\times 2=108$ in the box to make the equation true.
Evaluating the five choices, we obtain
(A) 48 (B) 72 (C) 108 (D) 64 (E) 432
Therefore, the answer must be $2\times 6\times 9$. Answer: (C)

8. If we turn the words

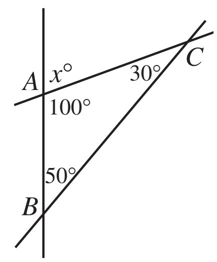
PUNK CD FOR SALE

around and look at them as if looking at them through the opposite side of a window, the only letters that appear the same will be U, O, and A. The other letters will all appear differently from the other side of the window. Answer: (A)

9. Spencer starts 1000 m from home and walks to a point 800 m from home, a distance of 200 m. He then walks to a point 1000 m from home, for a distance of another 200 m. Finally, he walks home, a distance of 1000 m. So Spencer has walked a total of $200+200+1000=1400$ m.

ANSWER: (E)

10. Since the sum of the angles in a triangle is $180^\circ$, then $\angle BAC = 180^\circ - 30^\circ - 50^\circ = 100^\circ$.
Since a straight line makes an angle of $180^\circ$, then $x^\circ + 100^\circ = 180^\circ$ or $x = 80$.




ANSWER: (A)

## Part B

11. Since there are 12 squares initially, then the number of squares to be removed is
$\frac{1}{2}\times\frac{2}{3}\times 12 = \frac{1}{3}\times 12 = 4$
Therefore, there will be 8 squares remaining.

ANSWER: (D)

12. *Solution 1*
Since the perimeter of the field is 3 times the length and the perimeter is 240 m, then the length of the field is 80 m.
Since the perimeter of a rectangle is two times the length plus two times the width, then the length accounts for 160 m of the perimeter, leaving 80 m for two times the width.
Therefore, the width of the field is 40 m.

*Solution 2*
Let the perimeter of the field be $P$, the length be $l$, and the width $w$.
We are given that $P = 3l$ and $P = 240$, so $l = \frac{1}{3}(240) = 80$.
Since $P = 240$ and $P = 2l + 2w$, we have $240 = 2(80) + 2w$ or $w = 40$.

ANSWER: (B)

13. Since Chris runs $\frac{1}{2}$ as fast as his usual running speed, he runs at 5 km/h, and so will take 6 hours to complete the 30 km run.
Since Pat runs at $1\frac{1}{2}$ her usual running speed, she runs at 15 km/h, and so will take 2 hours to complete the 30 km run.
Thus, it takes Chris 4 hours longer to complete the run than it takes Pat.

ANSWER: (D)

14. *Solution 1*

Since there are twice as many red disks as green disks and twice as many green disks as blue disks, then there are four times as many red disks as blue disks.
So the total number of disks is seven times the number of blue disks (since the numbers of red and green disks are four and two times the number of blue disks).
Since there are 14 disks in total, there are 2 blue disks, and so there are 4 green disks.

*Solution 2*

Let the number of green disks be $g$.
Then the number of red disks is $2g$, and the number of blue disks is $\frac{1}{2}g$.
From the information given,

$$\begin{aligned} 2g + g + \tfrac{1}{2}g &= 14 \\ \tfrac{4}{2}g + \tfrac{2}{2}g + \tfrac{1}{2}g &= 14 \\ \tfrac{7}{2}g &= 14 \\ g &= \tfrac{2}{7}\times 14 \\ g &= 4 \end{aligned}$$

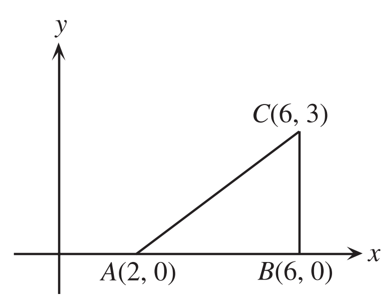
Therefore, the number of green disks is 4. ANSWER: (B)

15. In the bottle, there are a total of 180 tablets.
Among the 60 stars, there are an equal number of each of the three flavours – strawberry, grape and orange. This tells us that there are 20 grape stars.
If each tablet is equally likely to be chosen from the bottle, the probability of choosing a grape star is

$$\frac{\text{Number of grape stars}}{\text{Total number of tablets}} = \frac{20}{180} = \frac{1}{9}$$

ANSWER: (A)

16. First, we sketch the triangle.
Since point $C$ is directly above point $B$ (that is, angle $ABC$ is a right angle), then we can look at triangle $ABC$ as having base $AB$ (of length 4) and height $BC$ (of length 3).
Thus, the area of the triangle is $A = \frac{1}{2}bh = \frac{1}{2}(4)(3) = 6$ square units.




ANSWER: (C)

17. If Genna's total bill was \$74.16 and a \$45 fee was charged, this means that her cost based on the distance she drove was $\$74.16 - \$45.00 = \$29.16$. Thus the number of kilometres driven is $\frac{29.16}{0.12} = 243$.
ANSWER: (C)

18. *Solution 1*

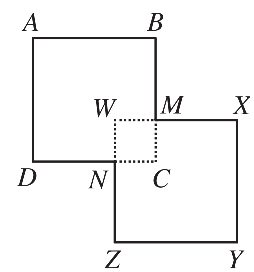
We can calculate the perimeter of the shaded figure by adding the perimeters of the two large squares and subtracting the perimeter of the small square. This is because all of the edges of the two larger squares are included in the perimeter of the shaded figure except for the sides of the smaller square. Since the side length of the larger squares is 5 cm, the perimeter of each of the two larger squares is 20 cm.

Since the area of the smaller square is $4\text{ cm}^2$, then its side length is 2 cm, and so its perimeter is 8 cm. Therefore, the perimeter of the shaded figure is $20 + 20 - 8 = 32$ cm.

*Solution 2*

We label the vertices in the diagram and calculate directly.

We know that the two large squares each have a side length of 5 cm, so $AB = AD = YX = YZ = 5$.




The smaller square has an area of $4\text{ cm}^2$, and so has a side length of 2 cm. Therefore, $WM = MC = CN = NW = 2$, and so each of $BM$, $DN$, $MX$, and $NZ$ have a length of 3 cm (the difference between the side length of the two squares).

Thus, the total perimeter of the shaded figure is $4 \times 5 + 4 \times 3 = 32$ cm.

ANSWER: (B)

19. Abraham's exam had a total of 80 questions. Since he received a mark of 80%, he got $\frac{80}{100} \times 80 = \frac{8}{10} \times 80 = 64$ questions correct.

We also know that Abraham answered 70% of the 30 algebra questions correctly, or a total of 21 questions.

This tells us that he answered 43 of the geometry questions correctly. ANSWER: (A)

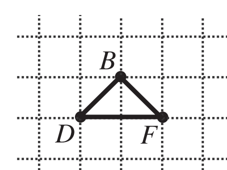
20. We first make a list of all of the possible triangles with an edge lying on *DEF* (that is, with an edge that is one of *DE*, *DF* or *EF*):

*DAE*, *DBE*, *DCE*, *DAF*, *DBF*, *DCF*, *EAF*, *EBF*, *ECF*

Of these, triangles *DAE*, *DBE*, *DAF*, *DCF*, *EBF*, *ECF* are obviously right-angled, while triangles *DCE* and *EAF* are not right-angled because they each contain an angle of $135^\circ$. What about triangle *DBF*?

In fact, triangle *DBF* is right-angled since $\angle DBE = \angle EBF = 45^\circ$, and so $\angle DBF = 90^\circ$.

Therefore, there are 2 triangles among this group which are not right-angled.




If we now look at the possible triangles with an edge lying on *ABC*, we will again find 2 non right-angled triangles among these triangles. (We can see this by reflecting all of the earlier triangles to move their base from *DEF* to *ABC*.)

We can also see that it is impossible to make a triangle without an edge lying on *ABC* or *DEF*, since the only other possible edges are *AD*, *AE*, *AF*, *BD*, *BE*, *BF*, *CD*, *CE*, and *CF*. No three of these nine edges can be joined to form a triangle since each edge has one end at *A*, *B* or *C* and the other end at *D*, *E* or *F*, and there are no edges joining two points among either set of three points.

Therefore, there are only 4 triangles that can be formed that are not right-angled.

ANSWER: (E)

## Part C

21. Since there are ten people scheduled for an operation and each operation begins 15 minutes after the previous one, the tenth operation will begin nine 15 minute intervals after the first operation began at 8:00 a.m.
Nine 15 minute intervals is 135 minutes, or 2 hours 15 minutes.
Thus, the tenth 45 minute operation begins at 10:15 a.m. and so ends at 11:00 a.m.

ANSWER: (D)

22. *Solution 1*

Since Luke has a 95% winning percentage, then he hasn't won 5%, or $\frac{1}{20}$, of his games to date. Since he has played only 20 games, there has only been 1 game that he has not won.

For Luke to have exactly a 96% winning percentage, he must not have won 4%, or $\frac{1}{25}$, of his games. Since he wins every game between these two positions, when he has the 96% winning percentage, he has still not won only 1 game. Therefore, he must have played 25 games in total, or 5 more than initially.

*Solution 2*

Since Luke has played 20 games and has a 95% winning percentage, then he has won $\frac{95}{100}\times 20 = \frac{95}{5} = 19$ games.

Let the number of games in a row he wins before reaching the 96% winning percentage be $x$. Then

$$\text{Winning }\% = \frac{\text{Games Won}}{\text{Games Played}} = \frac{19+x}{20+x} = \frac{96}{100}$$

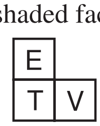
Cross-multiplying,

$$\begin{aligned} 100(19+x) &= 96(20+x) \\ 1900+100x &= 1920+96x \\ 4x &= 20 \\ x &= 5 \end{aligned}$$

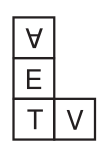
Therefore, we wins 5 more games in a row.

ANSWER: (D)

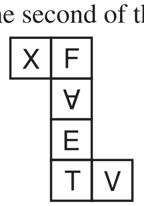
23. We will determine which letter belongs on the shaded face by "unfolding" the cube.
Using the first of the three positions, we obtain




Using the third of the three positions, we can add the "A" above the "E" as




Using the second of the three positions, the F can be placed above the A and the X to the left of the F to get



If we refold this cube, with A at the front, F on top, and E on the bottom, the right-hand face will be the V (upside down), and so the shaded face is the V. ANSWER: (E)

24. To get a better understanding of the pattern, let us write each of the numbers in the way that it is obtained:

$$\begin{array}{ccccccccc} & & & 1 & & 2 & & & \\ & & 1 & & 3 & & 2 & & \\ & 1 & & 4 & & 5 & & 2 & \\ 1 & & 5 & & 9 & & 7 & & 2 \\ \iddots & & & & \vdots & & & & \ddots \end{array} \qquad \begin{array}{ccccccccc} & & & 1 & & 2 & & & \\ & & 1 & & 1+2 & & 2 & & \\ & 1 & & 1+3 & & 3+2 & & 2 & \\ 1 & & 1+4 & & 4+5 & & 5+2 & & 2 \\ \iddots & & & & \vdots & & & & \ddots \end{array}$$

We can see from these two patterns side by side that each number in a row is accounted for *twice* in the row below. (The 1 or 2 on the end of a row appears again at the end of the row and as part of the sum in one number in the next row. A number in the middle of a row appears as part of the sum in two numbers in the next row.)
Therefore, the sum of the numbers in a row should be two times the sum of the numbers in the previous row.
We can check this:

| | |
|---|---|
| Sum of the numbers in the 1st row | 3 |
| Sum of the numbers in the 2nd row | 6 |
| Sum of the numbers in the 3rd row | 12 |
| Sum of the numbers in the 4th row | 24 |

Thus, the sum of the numbers in the thirteenth row should be the sum of the elements in the first row multiplied by 2 twelve times, or $3 \times 2^{12} = 3 \times 4096 = 12\,288$. ANSWER: (D)

25. We will present a complete consideration of all of the cases. The answer can be obtained more easily in a trial and error fashion.

First, we rewrite the equation putting letters in each of the boxes

$$\begin{array}{ccc} & A & B \\ & \times & C \\ \hline D & E & F \end{array}$$

We want to replace $A$, $B$, $C$, $D$, $E$, and $F$ by the digits 1 through 6.

Could $C$ be 1?

If $C$ was 1, then $B$ and $F$ would have to be same digit, which is impossible since all of the digits are different. Therefore, $C$ cannot be 1.

Could $C$ be 5?

If $C$ was 5, then if $B$ were odd, $F$ would also be 5, which would be impossible. If $C$ was 5 and $B$ even, then $F$ would have to be 0, which is also impossible. Therefore, $C$ cannot be 5.

Could $C$ be 6?

If $C$ was 6, let us list the possibilities for $B$ and the resulting value of $F$:

| $B$ | $F$ | |
|---|---|---|
| 1 | 6 | Impossible – two 6's |
| 2 | 2 | Impossible – two 2's |
| 3 | 8 | Impossible – no 8 |
| 4 | 4 | Impossible – two 4's |
| 5 | 0 | Impossible – no 0 |

Therefore, $C$ cannot be 6.

Could $C$ be 4?

If $C$ was 4, using a similar chart, we can see that $B$ must be 3 and $F$ must be 2. We will consider this possibility later.

Could $C$ be 3?

If $C$ was 4, using a similar chart, we can see that $B$ must be 2 or 4 and $F$ must be thus 6 or 2.

Could $C$ be 2?

If $C$ was 2, using a similar chart, we can see that $B$ must be 3 and $F$ must be 6.

Let us consider the case of $C = 2$. In this case, we have

$$\begin{array}{rrr} & A & 3 \\ & \times & 2 \\ \hline D & E & 6 \end{array}$$

Since the product has three digits, and the possibilities remaining for $A$ are 1, 4 or 5, then $A$ must be 5. However, this gives a product of $53 \times 2 = 106$, which is impossible.

Similarly, trying the case of $C = 4$, we are left with the possibilities for $A$ being 1, 5, or 6, none of which work.

Therefore, $C$ must be 3, since this is the only possibility left. We should probably check that we can actually get the multiplication to work, though!

Trying the possibilities as above, we can eventually see that $54 \times 3 = 162$ works, and so $C = 3$.

Answer: (B)