



# Canadian Mathematics Competition

An activity of the Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

## Fermat Contest (Grade 11)

Tuesday, February 20, 2007

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**Time:** 60 minutes

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**Calculators are permitted**

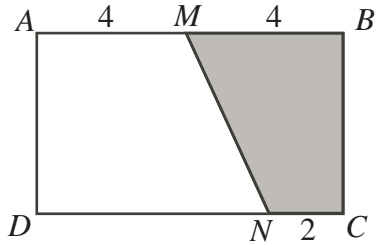
### Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E.** Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

The names of some top-scoring students will be published in the PCF Results on our Web site,  
<http://www.cemc.uwaterloo.ca>.

Scoring: There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

**Part A: Each correct answer is worth 5.**

1. The value of  $\frac{36 - 12}{12 - 4}$  is  
(A) 6                      (B) 9                      (C) 1                      (D) 31                      (E) 3
  2. If  $7x = 28$  and  $x + w = 9$ , what is the value of  $xw$ ?  
(A) 9                      (B) 20                      (C) 18                      (D) 52                      (E)  $-252$
  3. Of the fractions  $\frac{3}{4}$ ,  $\frac{7}{8}$ ,  $\frac{13}{16}$ , and  $\frac{1}{2}$ , what is the difference between the largest and the smallest?  
(A)  $\frac{3}{8}$                       (B)  $\frac{6}{7}$                       (C)  $\frac{5}{16}$                       (D)  $\frac{1}{16}$                       (E)  $\frac{1}{8}$
  4. When  $x = -5$ , the value of  $-2x^2 + \frac{5}{x}$  is  
(A) 99                      (B) 101                      (C)  $-51$                       (D) 19                      (E)  $-49$
  5. What is the value of  $1^{-2} + 2^{-1}$ ?  
(A)  $\frac{3}{2}$                       (B)  $\frac{1}{27}$                       (C) 4                      (D)  $-4$                       (E) 9
  6. In the diagram, the area of rectangle  $ABCD$  is 40. The area of  $MBCN$  is  
(A) 15                      (B) 10                      (C) 30  
(D) 12                      (E) 16
- 

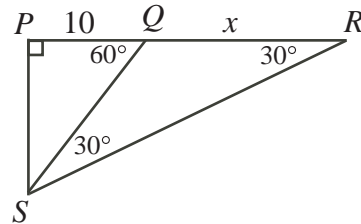
The diagram shows a rectangle  $ABCD$  with vertices  $A$  (top-left),  $B$  (top-right),  $C$  (bottom-right), and  $D$  (bottom-left). Point  $M$  is on side  $AB$  such that  $AM = 4$  and  $MB = 4$ . Point  $N$  is on side  $DC$  such that  $NC = 2$ . A line segment connects  $M$  and  $N$ , forming a shaded triangle  $MBCN$ .
7. The product of three positive integers is 42. The sum of two of these integers is 9. The third integer is  
(A) 1                      (B) 7                      (C) 6                      (D) 3                      (E) 2
  8. Ivan trained for a cross-country meet.  
On Monday, he ran a certain distance.  
On Tuesday, he ran twice as far as he ran on Monday.  
On Wednesday, he ran half as far as he ran on Tuesday.  
On Thursday, he ran half as far as he ran on Wednesday.  
On Friday, he ran twice as far as he ran on Thursday.  
If the shortest distance that he ran on any of the five days is 5 km, how far did he run in total?  
(A) 55 km                      (B) 25 km                      (C) 27.5 km                      (D) 17.5 km                      (E) 50 km
  9. If  $\frac{1}{x + 3} = 2$ , then the value of  $\frac{1}{x + 5}$  is  
(A)  $\frac{1}{2}$                       (B)  $\frac{2}{3}$                       (C)  $\frac{2}{5}$                       (D)  $\frac{1}{4}$                       (E) 4

10. A store normally sells each of its DVDs for \$20. At a sale, Phyllis buys two DVDs at the regular price and gets a third DVD for half price. This is the same rate of discount as getting
- (A) 2 for the price of 1  
 (B) 3 for the price of 2  
 (C) 4 for the price of 3  
 (D) 5 for the price of 4  
 (E) 6 for the price of 5

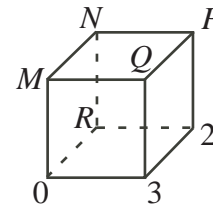
**Part B: Each correct answer is worth 6.**

11. Five numbers in increasing order are 2, 5,  $x$ , 10, and  $y$ . The median of the numbers is 7 and the mean (average) is 8. The value of  $y$  is
- (A) 16            (B) 14            (C) 15            (D) 18            (E) 12

12. In the diagram,  $PQ = 10$  and  $QR = x$ . The value of  $x$  is
- (A)  $10\sqrt{3}$       (B) 20            (C)  $\frac{50}{3}$   
 (D)  $\frac{20}{\sqrt{3}}$         (E) 10

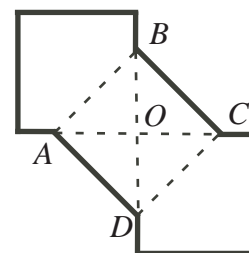


13. In the diagram, each of the numbers 0, 1, 2, 3, 4, 5, 6, and 7 is to be used to label a vertex of the cube. The numbers 0, 2 and 3 are placed as shown. The sum of the numbers at the ends of each edge must be a prime number. (Note: 1 is not a prime number.) The value of  $M + N + P + Q$  must be
- (A) 16            (B) 17            (C) 18  
 (D) 19            (E) 22



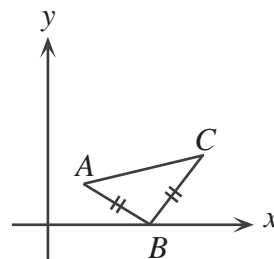
14. Two positive integers  $a$  and  $b$  have the property that if  $a$  is increased by 25%, the result will be *greater than* five times the value of  $b$ . What is the minimum possible value for  $a + b$ ?
- (A) 3            (B) 6            (C) 10            (D) 9            (E) 21
15. How many three-digit positive integers  $x$  are there with the property that  $x$  and  $2x$  have only even digits? (One such number is  $x = 420$ , since  $2x = 840$  and each of  $x$  and  $2x$  has only even digits.)
- (A) 64            (B) 18            (C) 16            (D) 125            (E) 100

16. In the diagram, each of the three squares has a side length of 3. Two of the squares have a common vertex  $O$ , and  $O$  is the centre of the square labelled  $ABCD$ . The perimeter of the entire figure is closest to
- (A) 21.5            (B) 22.0            (C) 22.5  
 (D) 24.0            (E) 30.0



17. In the diagram,  $A(2,2)$  and  $C(8,4)$  are two of the vertices of an isosceles right-angled triangle  $ABC$ . If the vertex  $B$  is located on the  $x$ -axis and  $\angle ABC = 90^\circ$ , the  $x$ -coordinate of  $B$  is

(A) 3                      (B) 4                      (C) 5  
 (D) 6                      (E) 7

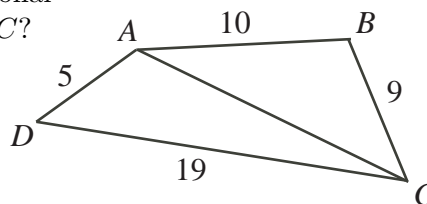


18. Alphonso and Karen started out with the *same number* of apples. Karen gave twelve of her apples to Alphonso. Next, Karen gave half of her remaining apples to Alphonso. If Alphonso now has four times as many apples as Karen, how many apples does Karen now have?

(A) 12                      (B) 24                      (C) 36                      (D) 48                      (E) 72

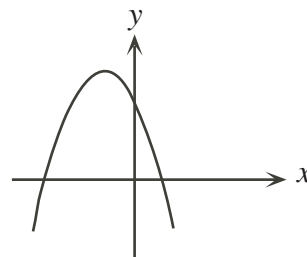
19. In the diagram,  $ABCD$  is a quadrilateral with diagonal  $AC$ . Which of the following is a possible length for  $AC$ ?

(A) 9                      (B) 10                      (C) 13  
 (D) 15                      (E) 20



20. The graph of the function  $y = ax^2 + bx + c$  is shown in the diagram. Which of the following must be positive?

(A)  $a$                       (B)  $bc$                       (C)  $ab^2$   
 (D)  $b - c$                       (E)  $c - a$



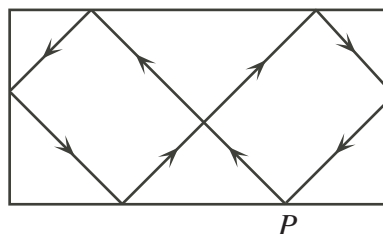
**Part C: Each correct answer is worth 8.**

21. Five consecutive positive integers have the property that the sum of the second, third and fourth is a perfect square, while the sum of all five is a perfect cube. If  $m$  is the third of these five integers, then the minimum possible value of  $m$  satisfies

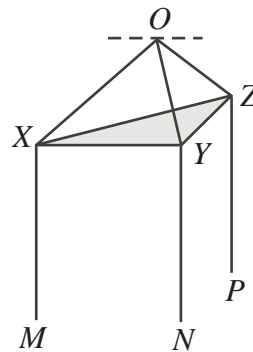
(A)  $m \leq 200$   
 (B)  $200 < m \leq 400$   
 (C)  $400 < m \leq 600$   
 (D)  $600 < m \leq 800$   
 (E)  $m > 800$

22. A ball placed at point  $P$  on a rectangular billiard table is shot at an angle of  $45^\circ$  to the edge of the table. After successively bouncing off the edges of the table at  $45^\circ$  angles, it returns to point  $P$ , as shown. If the ball travels 7 m, the perimeter, in metres, of the table is closest to

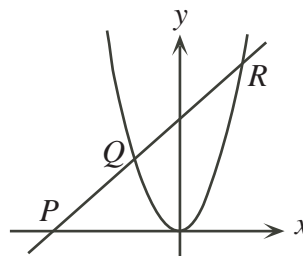
(A) 7.0                      (B) 7.5                      (C) 8.0  
 (D) 8.5                      (E) 9.0



23. An ugly light fixture is hanging from point  $O$  on the ceiling. Wires  $OXM$ ,  $OYN$  and  $OZP$  pass through the vertices of a very thin wooden equilateral triangle  $XYZ$  of side 60 cm. (A small bulb is attached to the end of each wire.) The plane of the wooden triangle is parallel to the ceiling. If each wire is 100 cm long and the lower end of each wire is 90 cm from the ceiling, what is the vertical distance between the wooden triangle and the ceiling?
- (A) 40 cm      (B) 45 cm      (C) 50 cm  
 (D) 55 cm      (E) 60 cm



24. A line with slope 1 passes through point  $P$  on the negative  $x$ -axis and intersects the parabola  $y = x^2$  at points  $Q$  and  $R$ , as shown. If  $PQ = QR$ , then the  $y$ -intercept of  $PR$  is closest to
- (A) 9.9      (B) 10.2      (C) 8.2  
 (D) 9.3      (E) 8.6



25. How many ordered pairs  $(b, g)$  of positive integers with  $4 \leq b \leq g \leq 2007$  are there such that when  $b$  black balls and  $g$  gold balls are randomly arranged in a row, the probability that the balls on each end have the same colour is  $\frac{1}{2}$ ?
- (A) 60      (B) 62      (C) 58      (D) 61      (E) 59



## Canadian Mathematics Competition



### *For students...*

Thank you for writing the 2007 Fermat Contest!  
In 2006, more than 90 000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

Encourage your teacher to register you for Hypatia Contest which will be written on April 18, 2007.

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