2009 Galois Contest (Grade 10)

Wednesday, April 8, 2009

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1. Alex counts the number of students in her class with each hair colour, and summarizes the results in the following table:

Hair Colour	Number of Students
Blonde	8
Brown	7
Red	3
Black	2

   (a) What percentage of students in the class have blonde hair?

   (b) What percentage of students in the class have red or black hair?

   (c) How many students in the class with blonde hair would have to dye their hair black for the percentage of students in the class with black hair to be 20%?

   (d) How many students with red hair would have to join the class so the percentage of students in the class with red hair is equal to 32%?

2. A square has vertices with coordinates $A(6, 9)$, $B(12, 6)$, $C(t, 0)$, and $D(3, 3)$.

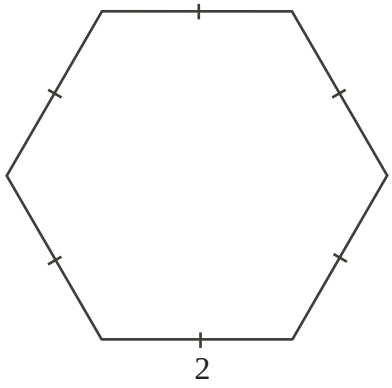
   (a) Determine the value of t, the x-coordinate of vertex C.

   (b) A line is drawn through $O(0, 0)$ and D. This line meets AB at E. Determine the coordinates of E.

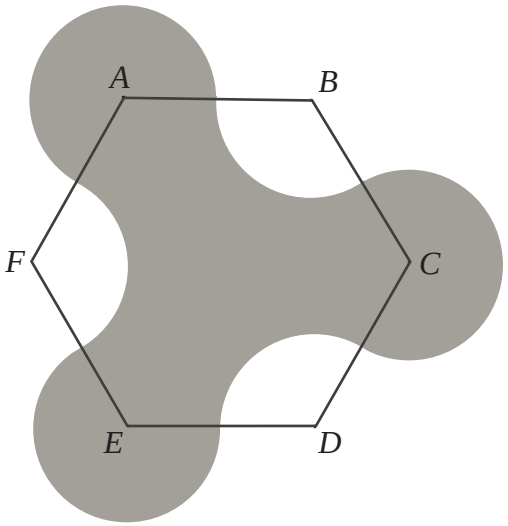
   (c) Determine the perimeter of quadrilateral $EBCD$.

3. (a) Find the area of an equilateral triangle with side length 2.

   (b) Determine the area of a regular hexagon with side length 2.



(c) In the diagram, regular hexagon $ABCDEF$ has sides of length 2. Using A, C and E as centres, portions of circles with radius 1 are drawn outside the hexagon. Using B, D and F as centres, portions of circles with radius 1 are drawn inside the hexagon. These six circular arcs join together to form a curve. Determine the area of the shaded region enclosed by this curve.



4. If m is a positive integer, the symbol $m!$ is used to represent the product of the integers from 1 to m. That is, $m! = m(m-1)(m-2)\cdots(3)(2)(1)$. For example, $5! = 5(4)(3)(2)(1)$ or $5! = 120$.
Some positive integers n can be written in the form

$$n = a(1!) + b(2!) + c(3!) + d(4!) + e(5!).$$

In addition, each of the following conditions is satisfied:

- a, b, c, d, and e are integers
- $0 \leq a \leq 1$
- $0 \leq b \leq 2$
- $0 \leq c \leq 3$
- $0 \leq d \leq 4$
- $0 \leq e \leq 5$.

(a) Determine the largest positive integer N that can be written in this form.

(b) Write $n = 653$ in this form.

(c) Prove that all integers n, where $0 \leq n \leq N$, can be written in this form.

(d) Determine the sum of all integers n that can be written in this form with $c = 0$.