



The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

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Fermat Contest

(Grade 11)

Thursday, February 24, 2011

UNIVERSITY OF
WATERLOO

WATERLOO
MATHEMATICS

THE
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Time: 60 minutes

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Calculators are permitted

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D,** and **E.** Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

The names of some top-scoring students will be published in the PCF Results on our Web site, <http://www.cemc.uwaterloo.ca>.

Scoring: There is *no penalty* for an incorrect answer.
 Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. $\frac{2 + 3 \times 6}{23 + 6}$ is equal to

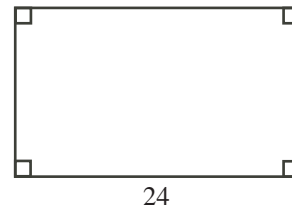
- (A) 1 (B) $\frac{11}{29}$ (C) $\frac{36}{29}$ (D) $\frac{20}{29}$ (E) $\frac{5}{23}$

2. If $y = 77$, then $\frac{7y + 77}{77}$ is equal to

- (A) 8 (B) 12 (C) 78 (D) 84 (E) 540

3. The area of the rectangle shown is 192. What is the perimeter of the rectangle?

- (A) 64 (B) 384 (C) 192
 (D) 1728 (E) 32

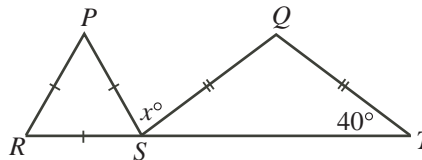


4. If $\sqrt{n + 9} = 25$, then n equals

- (A) 256 (B) -4 (C) 484 (D) 616 (E) 16

5. In the diagram, S lies on RT , $\angle QTS = 40^\circ$, $QS = QT$, and $\triangle PRS$ is equilateral. The value of x is

- (A) 50 (B) 60 (C) 80
 (D) 90 (E) 100



6. When three consecutive integers are added, the total is 27. When the same three integers are multiplied, the result is

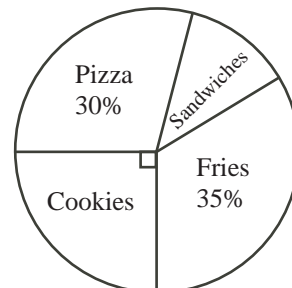
- (A) 504 (B) 81 (C) 720 (D) 729 (E) 990

7. The number halfway between $\frac{1}{12}$ and $\frac{1}{10}$ is

- (A) $\frac{1}{11}$ (B) $\frac{1}{120}$ (C) $\frac{11}{60}$ (D) $\frac{11}{120}$ (E) $\frac{1}{22}$

8. The circle graph shown illustrates the results of a survey taken by the Fermat H.S. Student Council to determine the favourite cafeteria food. How many of the 200 students surveyed said that their favourite food was sandwiches?

- (A) 10 (B) 20 (C) 35
 (D) 50 (E) 70

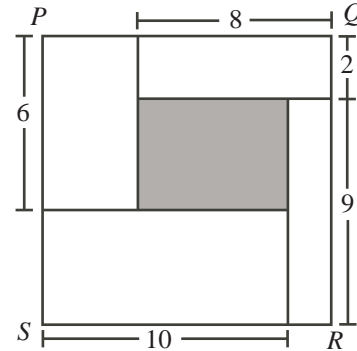


9. The set $S = \{1, 2, 3, \dots, 49, 50\}$ contains the first 50 positive integers. After the multiples of 2 and the multiples of 3 are removed, how many numbers remain in the set S ?

(A) 8 (B) 9 (C) 16 (D) 17 (E) 18

10. In the diagram, $PQRS$ is a square. Square $PQRS$ is divided into five rectangles, as shown. The area of the shaded rectangle is

(A) 49 (B) 28 (C) 22
(D) 57 (E) 16



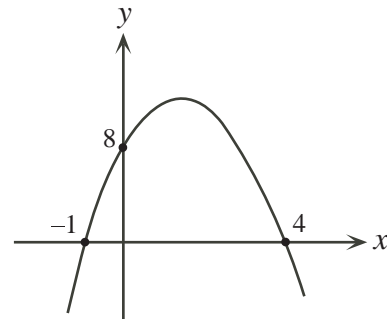
Part B: Each correct answer is worth 6.

11. A gumball machine that randomly dispenses one gumball at a time contains 13 red, 5 blue, 1 white, and 9 green gumballs. What is the least number of gumballs that Wally must buy to *guarantee* that he receives 3 gumballs of the same colour?

(A) 6 (B) 9 (C) 4 (D) 7 (E) 8

12. In the diagram, the parabola has x -intercepts -1 and 4 , and y -intercept 8 . If the parabola passes through the point $(3, w)$, what is the value of w ?

(A) 4 (B) 5 (C) 6
(D) 7 (E) 8



13. Xavier, Yolanda, and Zixuan have a total of \$50. The ratio of the amount Xavier has to the total amount Yolanda and Zixuan have is $3 : 2$. Yolanda has \$4 more than Zixuan. How much does Zixuan have?

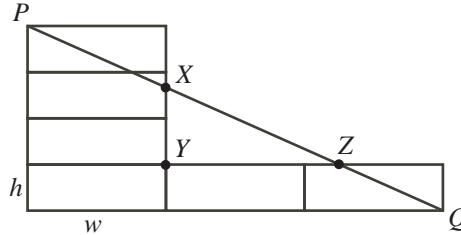
(A) \$16 (B) \$8 (C) \$14 (D) \$13 (E) \$30

14. Which of the following *must* be an even integer?

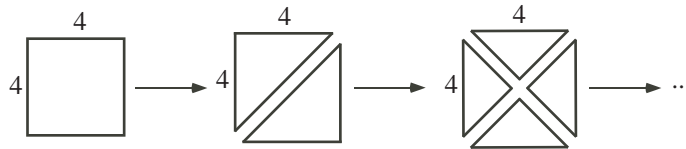
(A) The average of two even integers
(B) The average of two prime numbers
(C) The average of two perfect squares
(D) The average of two multiples of 4
(E) The average of three consecutive integers

15. If m and n are consecutive positive integers and $n^2 - m^2 > 20$, then the minimum possible value of $n^2 + m^2$ is
- (A) 29 (B) 181 (C) 265 (D) 23 (E) 221

16. Six identical rectangles with height h and width w are arranged as shown. Line segment PQ intersects the vertical side of one rectangle at X and the horizontal side of another rectangle at Z . If right-angled $\triangle XYZ$ has $YZ = 2XY$, then $\frac{h}{w}$ equals

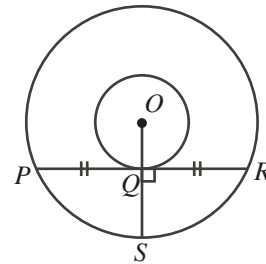


- (A) $\frac{2}{3}$ (B) $\frac{1}{2}$ (C) $\frac{3}{8}$
 (D) $\frac{1}{3}$ (E) $\frac{3}{4}$
17. If $3^{2x} = 64$, then 3^{-x} is equal to
- (A) -32 (B) -8 (C) $\frac{1}{4096}$ (D) $\frac{1}{32}$ (E) $\frac{1}{8}$
18. A 4×4 square piece of paper is cut into two identical pieces along its diagonal. The resulting triangular pieces of paper are each cut into two identical pieces.



Each of the four resulting pieces is cut into two identical pieces. Each of the eight new resulting pieces is finally cut into two identical pieces. The length of the longest edge of one of these final sixteen pieces of paper is

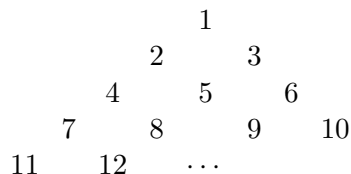
- (A) 1 (B) 2 (C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$ (E) $2\sqrt{2}$
19. In the diagram, the two circles are centred at O . Point S is on the larger circle. Point Q is the point of intersection of OS and the smaller circle. Line segment PR is a chord of the larger circle and touches (that is, is tangent to) the smaller circle at Q . Note that OS is the perpendicular bisector of PR . If $PR = 12$ and $QS = 4$, then the radius of the larger circle is



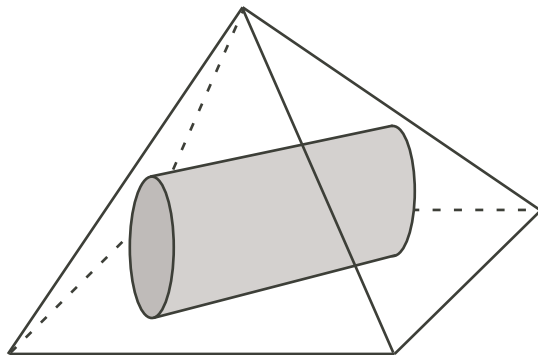
- (A) 6.0 (B) 5.0 (C) 6.5
 (D) 7.2 (E) 20.0
20. Three real numbers a, b and c have a sum of 114 and a product of 46 656. If $b = ar$ and $c = ar^2$ for some real number r , then the value of $a + c$ is
- (A) 78 (B) 76 (C) 24 (D) 54 (E) 36

Part C: Each correct answer is worth 8.

21. The positive integers are arranged in increasing order in a triangle, as shown. Each row contains one more number than the previous row. The sum of the numbers in the row that contains the number 400 is



- (A) 10 990 (B) 12 209 (C) 9855
(D) 10 976 (E) 11 368
22. The number of pairs of positive integers (p, q) , with $p + q \leq 100$, that satisfy the equation $\frac{p + q^{-1}}{p^{-1} + q} = 17$ is
- (A) 0 (B) 1 (C) 2 (D) 4 (E) 5
23. Dolly, Molly and Polly each can walk at 6 km/h. Their one motorcycle, which travels at 90 km/h, can accommodate at most two of them at once (and cannot drive by itself!). Let t hours be the time taken for all three of them to reach a point 135 km away. Ignoring the time required to start, stop or change directions, what is true about the smallest possible value of t ?
- (A) $t < 3.9$ (B) $3.9 \leq t < 4.1$ (C) $4.1 \leq t < 4.3$
(D) $4.3 \leq t < 4.5$ (E) $4.5 \leq t$
24. Four numbers w, x, y, z satisfy $w < x < y < z$. Each of the six possible pairs of distinct numbers has a different sum. The four smallest sums are 1, 2, 3, and 4. What is the sum of all possible values of z ?
- (A) 4 (B) $\frac{13}{2}$ (C) $\frac{17}{2}$ (D) $\frac{15}{2}$ (E) 7
25. A pyramid has a square base with side length 20. A right circular cylinder has a diameter of 10 and a length of 10. The cylinder is lying on its side, completely inside the pyramid. The central axis of the cylinder lies parallel to and directly above a diagonal of the pyramid's base. The midpoint of the central axis lies directly above the centre of the square base of the pyramid.



The smallest possible height of the pyramid is closest to

- (A) 15.3 (B) 22.1 (C) 21.9 (D) 21.7 (E) 15.5



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