The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Galois Contest
(Grade 10)

Wednesday, April 10, 2019
(in North America and South America)

Thursday, April 11, 2019
(outside of North America and South America)

Time: 75 minutes

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Do not open this booklet until instructed to do so.

Number of questions: 4 Each question is worth 10 marks

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Parts of each question can be of two types:

1. SHORT ANSWER parts indicated by •
   - worth 2 or 3 marks each
   - full marks given for a correct answer which is placed in the box
   - part marks awarded only if relevant work is shown in the space provided

2. FULL SOLUTION parts indicated by
   - worth the remainder of the 10 marks for the question
   - must be written in the appropriate location in the answer booklet
   - marks awarded for completeness, clarity, and style of presentation
   - a correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.
1. The Galois Restaurant is in a region that adds 10% sales tax onto the price of food and drinks purchased at a restaurant. The prices listed on their menu do not include the sales tax.

(a) From the menu, Becky orders a plate of lasagna listed for $7.50, a side salad listed for $5.00, and a lemonade listed for $3.00. After tax is included, how much is Becky’s total bill?

(b) A burrito is listed on the menu for $6.00. After tax is included, what is the greatest number of burritos that Jackson can buy if he has $50.00?

(c) On the Galois Restaurant menu, hotdogs are listed at the regular price of $5.00. The restaurant has the following promotional deals:

- On Mondays, if you buy a hotdog at the regular menu price of $5.00, then the price for a second hotdog is $4.50.

- On Tuesdays, you pay half the tax on all hotdogs.

Chase bought two hotdogs on Monday and then two hotdogs on Tuesday. After tax is included, determine on which day Chase spent less money.
2. The hypotenuse of right-angled \( \triangle AOB \) lies on the line with equation \( y = -2x + 12 \), as shown in Figure 1. The legs of \( \triangle AOB \) lie on the axes.

(a) What is the area of \( \triangle AOB \)?

(b) A second line passes through \( O \) and is perpendicular to the first line, as shown in Figure 2. The two lines intersect at \( C \). Determine the coordinates of \( C \).

(c) The second line passes through the point \( D \) in the first quadrant, as shown in Figure 3. Points \( E \) and \( F \) are positioned on the axes so that \( DEOF \) is a rectangle. If the area of \( DEOF \) is 1352, determine the coordinates of \( D \).

3. If \( n \) is a positive integer, the notation \( n! \) (read “\( n \) factorial”) is used to represent the product of the integers from 1 to \( n \). That is, \( n! = n(n-1)(n-2) \cdots (3)(2)(1) \). For example, \( 5! = 5(4)(3)(2)(1) \) or \( 5! = 120 \).

(a) What is the largest positive integer \( m \) for which \( 2^m \) is a divisor of \( 9! \) ?

(b) What is the smallest value of \( n \) for which \( n! \) is divisible by \( 7^2 \)?

(c) Explain why there is no positive integer \( n \) for which \( n! \) is divisible by \( 7^7 \) but is not divisible by \( 7^8 \).

(d) Show that there is exactly one positive integer \( n \) for which

\[ n! = 2^a \cdot 3^b \cdot 5^c \cdot 7^d \cdot 11^2 \cdot 13^2 \cdot 17 \cdot 19 \cdot 23, \]

and

\[ a + b + c + d = 45 \]

for some positive integers \( a, b, c, d \).

4. A positive integer is digit-balanced if each digit \( d \), with \( 0 \leq d \leq 9 \), appears at most \( d \) times in the integer. For example, 13224 is digit-balanced, but 21232 is not.

(a) Explain why a digit-balanced integer is not divisible by 10.

(b) How many 4-digit integers have all non-zero digits and are not digit-balanced?

(c) Determine all positive integers \( k \) for which there exist digit-balanced positive integers \( m \) and \( n \), where \( m + n = 10^k \) and \( m \) and \( n \) each have \( k \) digits.
For students...

Thank you for writing the 2019 Galois Contest! Each year, more than 260,000 students from more than 80 countries register to write the CEMC’s Contests.

Encourage your teacher to register you for the Canadian Intermediate Mathematics Contest or the Canadian Senior Mathematics Contest, which will be written in November 2019.

Visit our website cemc.uwaterloo.ca to find

- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website cemc.uwaterloo.ca to

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- Register your students for the Canadian Senior and Intermediate Mathematics Contests which will be written in November
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