

Answers to Practice Set Number 1

Pascal

1) E 2) E 3) D 4) C 5) D 6) B 7) A 8) E 9) D 10) C

Cayley

1) E 2) C 3) A 4) E 5) A 6) D 7) C 8) A 9) C 10) E

Fermat

1) C 2) B 3) C 4) E 5) C 6) C 7) B 8) D 9) C 10) E

Hints, suggestions, and some solutions:

Pascal

1. Think of 399 as $400 - 1$.
2. Consider that at each stage he retained $\frac{1}{4}$ of the money or alternately just work this problem backwards!
3. $\frac{4}{3} \times 96 = 128$
4. Since the first two vertices lie on a vertical line, the area depends only on the horizontal distance of the third vertex from this line!
5. Recall $1000 = 7 \times 11 \times 13$ is divisible by 7, as is $1001 - 7 = 994$. But our sequence is the set of numbers that leave a remainder of 4 when divided by 7. So which of these numbers leaves a remainder of 4 when divided by 7?
6. If the areas of the two fields are x and y then calculating the area devoted to tomatoes in two different ways gives $.65x + .54y = .53(x + y)$. After a little algebra you can find the ration $x : y = 2 : 3$. Therefore $x : (x + y) = 2.5$.
7. If we define the amount of work done by one worker for 1 day as 1 unit of work then the wall requires 1800 units. After 10 days 1500 units are still required. Thus to finish in another 30 days requires 50 workers, or 20 more than the original 30.
8. " BAD " = $100B + 10A + D$ etc. So the totals is $102D + 101M + 100B + 30A$. This is as large as possible when $D = 4$, $M = 3$, $B = 2$ and $A = 1$.
9. Count the horizontal and vertical blocks separately! In the tower of 80 stories we get $(1+2+3+\dots+80)$ as the number of horizontal blocks. Similarly the number of vertical blocks is just 80 more than the number of horizontal blocks. Use the formula for the sum $1 + 2 + 3 + \dots + n = n(n + 1)/2$.
10. The sum of the angles in any triangle is 180° . Also, the sum of angles along a straight line is 180° . Then $\angle BYA + 90^\circ + \angle GYF = 180^\circ$, $\angle BYA + 90^\circ + \angle YAB = 180^\circ$, and $\angle GYF = 90^\circ + \angle YFG = 180^\circ$. Since $AY = YF$, $\triangle ABY$ and $\triangle FYG$ are congruent triangles, and $BY = FG$.

Cayley

1. The simplest solution is to just clear fractions by multiplying numerator and denominator by 6.
2. There are 20 boards and 19 spaces so the total width in centimetres is $280 + 38 = 318 \text{ cm} = 3.18$ metres.
3. Maria's total mark on the first four tests is $90 \times 4 = 360$
4. Recall $A = \text{length} \times \text{Width}$. So factor the expression for the area. You will find the width is $2x - y$. This makes the perimeter equal to $2(2x - y) + 2(2x + y) = 4x - 2y + 4x + 2y = 8x$.
5. Calculate the slope! Using the slope of 5 you can see that $6 - b = 20$ and $b = -14$.
6. Square both sides, reorganize and square again!
7. The trapezoid is isosceles! After some work you can see that the diagonals are the hypotenuses of two $8 - 15 - 17$ triangles and so the altitude of the trapezoid is 8. Then calculate its area by dividing it into a rectangle and triangles.
8. $30030 = 30 \times 1001 = 2 \times 3 \times 5 \times 7 \times 11 \times 13$. So the three ages are 26, 33, 35.
9. If the hexagon has side 1, its area is $6 \left(\frac{\sqrt{3}}{4} \right)$. If the triangle ABC is divided into two $30 - 60 - 90$ triangles you can see $AC = \sqrt{3} = CE = AE$ and its area is $\left(\frac{\sqrt{3}}{4} \right) (\sqrt{3})^2 = 3 \frac{\sqrt{3}}{4}$ which is half the area of the hexagon.
10. After working with angles you will find that AXZ is equilateral and CXY and BZY are $30 - 60 - 90$. Since you know the ratios of all sides it is not too hard to find an equation for the required length.

Fermat

1. Since the sum is $5 \times 20 = 100$, the maximum is $100 - (1 + 2 + 3 + 4) = 90$.
2. Divide the area into a rectangle and a triangle!
3. Each of Peter's integers is 13 greater than the corresponding integer for Ian and so the sums differ by 169.
4. Only d and e are smaller than the original number which is c . Try experimenting with some numbers for x !
5. Add the equations to get $2(a + b + c) = 36$. So $a + b + c = 18$ and $a + b = 14$ which implies $c = 4$.
6. Introduce coordinates. Then the centers are at (r, r) and $(24, 29)$ and since the circles are tangent the distance between centers is $r + 16$. Thus we arrive at a factorable quadratic equation!
7. Can you explain why the other factor is $x^2 - 5x + 6$?

8. Since $x^2 = 8x + 13$, multiplying by x gives

$$x^3 = 8x^2 + 13x = 8(8x + 13) + 13x = 77x + 104$$

9. The piece removed is similar to the original cone and $\frac{1}{2}$ its size in all dimensions. Therefore the piece removed has $\frac{1}{2^3} = \frac{1}{8}$ its volume and the remaining pieces is $\frac{7}{8}$ of the original cone.
10. All such numbers are of the form pq or p^3 . A careful listing of the primes up to 47 will show that there are $30 + 2$ such numbers.