EXPERIENCE FIRST
Formalize Later

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<table>
<thead>
<tr>
<th>Experience</th>
<th>Formal Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Authentic to mathematical thinking (Not just being a student)</td>
<td>Terms and definitions</td>
</tr>
<tr>
<td>Everyone counting!</td>
<td>Symbols</td>
</tr>
<tr>
<td></td>
<td>Formulas</td>
</tr>
<tr>
<td></td>
<td>n(n-1) / 2</td>
</tr>
<tr>
<td></td>
<td>Factorial</td>
</tr>
<tr>
<td></td>
<td>Combinations,</td>
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<tr>
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<td>Permutations</td>
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</table>
The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.

### Conceptual Framework for K-9 Mathematics

**Nature of Mathematics**
- Change
- Constancy, Number Sense, Patterns, Relationships, Spatial Sense, Uncertainty

**Mathematical Processes:**
- Communication, Connections, Mental Mathematics and Estimation, Problem Solving, Reasoning, Technology, Visualization

**Strand**
- Number
- Patterns and Relations
  - Patterns
  - Variables and Equations
- Shape and Space
  - Measurement
  - 3-D Objects and 2-D Shapes
  - Transformations
- Statistics and Probability
  - Data Analysis
  - Chance and Uncertainty

**Grade**
- K 1 2 3 4 5 6 7 8 9

**General Learning Outcomes, Specific Learning Outcomes, and Achievement Indicators**
The Mathematical Processes

Students learn and apply the mathematical processes as they work to achieve the expectations outlined in the curriculum. All students are actively engaged in applying these processes throughout the program. They apply these processes, together with social-emotional learning (SEL) skills, across the curriculum to support learning in mathematics. See the section “Strand A: Social-Emotional Learning (SEL) Skills in Mathematics and the Mathematical Processes” for more information.

The mathematical processes that support effective learning in mathematics are as follows:

- problem solving
- reasoning and proving
- reflecting
- connecting
- communicating
- representing
- selecting tools and strategies
$10 + 10 + 8 + 8 = 36$
\[(8 \times 4) + 4 = 36\]
\[(10 \times 4) - 4 = 36\]
\[10 + 9 + 9 + 8 = 36\]
\[9 \times 4 = 36\]
\[(10 \times 10) - (8 \times 8) = 36\]
For 6 by 6

$(4 \times 4) + 4 = 20$

$(6 \times 4) - 4 = 20$

$6 + 5 + 5 + 4 = 20$

$5 \times 4 = 20$

$(6 \times 6) - (4 \times 4) = 20$

...
$n \times n$
<table>
<thead>
<tr>
<th>Grade 5</th>
<th>Grade 6</th>
<th>Grade 7</th>
<th>Grade 8</th>
<th>Grade 9</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Specific Learning Outcomes</strong></td>
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</tr>
<tr>
<td>5.PR.2. Solve problems involving single-variable (expressed as symbols or letters), one-step equations with whole-number coefficients, and whole-number solutions. [C, CN, PS, R]</td>
<td>6.PR.3. Represent generalizations arising from number relationships using equations with letter variables. [C, CN, PS, R, V]</td>
<td>7.PR.3. Demonstrate an understanding of preservation of equality by: - modelling preservation of equality, concretely, pictorially, and symbolically - applying preservation of equality to solve equations [C, CN, PS, R, V]</td>
<td>8.PR.2. Model and solve problems using linear equations of the form: - $ax = b$ - $\frac{x}{a} = b, a \neq 0$ - $ax + b = c$ - $\frac{x}{a} + b = c, a \neq 0$ - $a(x + b) = c$ concretely, pictorially, and symbolically, where $a$, $b$, and $c$ are integers. [C, CN, PS, V]</td>
<td>9.PR.3. Model and solve problems using linear equations of the form: - $ax = b$ - $\frac{x}{a} = b, a \neq 0$ - $ax + b = c$ - $\frac{x}{a} + b = c, a \neq 0$ - $ax = b + cx$ - $a(x + b) = cx + d$ - $a(bx + c) = dx + f$ - $\frac{x}{a} = b, x \neq 0$ where $a$, $b$, $c$, $d$, $e$, and $f$ are rational numbers. [C, CN, PS, V]</td>
</tr>
<tr>
<td>5.PR.4. Explain and illustrate strategies to solve single variable linear inequalities with rational number coefficients within a problem-solving context. [C, CN, PS, R]</td>
<td>6.PR.3. Demonstrate an understanding of preservation of equality by: - modelling preservation of equality, concretely, pictorially, and symbolically - applying preservation of equality to solve equations [C, CN, PS, R, V]</td>
<td>7.PR.4. Explain the difference between an expression and an equation. [C, CN]</td>
<td>8.PR.5. Demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2). [C, CN, R, V]</td>
<td>9.PR.4. Explain and illustrate strategies to solve single variable linear inequalities with rational number coefficients within a problem-solving context. [C, CN, PS, R]</td>
</tr>
</tbody>
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2.1 Introduction

In the earlier classes, you have come across several algebraic expressions and equations. Some examples of expressions we have so far worked with are:

\[ 5x, 2x - 3, 3x + y, 2xy + 5, xyz + x + y + z, x^2 + 1, y + y^2 \]

Some examples of equations are: \( 5x = 25, 2x - 3 = 9, 2y + \frac{5}{2} = \frac{37}{2}, 6z + 10 = -2 \)

You would remember that equations use the equality (=) sign; it is missing in expressions.

Of these given expressions, many have more than one variable. For example, \( 2xy + 5 \)
has two variables. We however, restrict to expressions with only one variable when we form equations. Moreover, the expressions we use to form equations are linear. This means that the highest power of the variable appearing in the expression is 1.

These are linear expressions:

\[ 2x, 2x + 1, 3y = 7, 12 = 5z, \frac{5}{4}(x - 4) + 10 \]

These are not linear expressions:

\[ x^2 + 1, y + y^2, 1 + z + z^2 + z^3 \] (since highest power of variable > 1)

Here we will deal with equations with linear expressions in one variable only. Such equations are known as linear equations in one variable. The simple equations which you studied in the earlier classes were all of this type.

Let us briefly revise what we know:

(a) An algebraic equation is an equality involving variables. It has an equality sign.

The expression on the left of the equality sign is the Left Hand Side (LHS). The expression on the right of the equality sign is the Right Hand Side (RHS).
What makes a good experience?
<table>
<thead>
<tr>
<th>CI For</th>
<th>Sample Statistic</th>
<th>Margin of Error</th>
<th>Use When</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population mean ($\mu$)</td>
<td>$\bar{x}$</td>
<td>$\pm z^* \frac{\sigma}{\sqrt{n}}$</td>
<td>$X$ is normal, or $n \geq 30$; $\sigma$ known</td>
</tr>
<tr>
<td>Population mean ($\mu$)</td>
<td>$\bar{x}$</td>
<td>$\pm t^*_{n-1} \frac{s}{\sqrt{n}}$</td>
<td>$n &lt; 30$, and/or $\sigma$ unknown</td>
</tr>
<tr>
<td>Population proportion ($p$)</td>
<td>$\hat{p}$</td>
<td>$\pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$</td>
<td>$np$, $n(1-\hat{p}) \geq 10$</td>
</tr>
<tr>
<td>Difference of two population means ($\mu_1 - \mu_2$)</td>
<td>$\bar{x}_1 - \bar{x}_2$</td>
<td>$\pm z^* \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$</td>
<td>Both normal distributions or $n_1, n_2 \geq 30$;</td>
</tr>
<tr>
<td>Difference of two population means $\mu_1 - \mu_2$</td>
<td>$\bar{x}_1 - \bar{x}_2$</td>
<td>$\pm t^*_{n_1+n_2-2} \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$</td>
<td>Both normal distributions or $n_1, n_2 \geq 30$;</td>
</tr>
<tr>
<td>Difference of two proportions ($p_1 - p_2$)</td>
<td>$\hat{p}_1 - \hat{p}_2$</td>
<td>$\pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$</td>
<td>Both normal distributions or $n_1, n_2 \geq 30$.</td>
</tr>
</tbody>
</table>
WHAT MAKES A GOOD EXPERIENCE?

- Accessible ways to derive formula
- Parallel structure
- Preview of inference
- Connecting content
- Giving students opportunity to use vocabulary
- Students create knowledge
- Immersion model
- Truly unknown answer
- Connecting lessons
- Class data
- Ask students to create
- Teacher not needed
- Allow opportunity for discussion
- Gradual release of responsibility
- Spiraled content
- Formalize condition

WHAT MAKES A GOOD EXPERIENCE?

- Allow students to gain confidence
- Require students to communicate
- Create a desire for a new tool
- Allow students to know the why
- Multiple solution paths
- Great context (hook) memoraible
- Fun, game
- Counterintuitive (surprise)
- Entry point is a good question
- Formalize later
- Thinking about margin
- Low risk
- Efficiency
- Anticipate student moves
- Concrete ↔ abstract
- Scaffolded
- Planting seeds
What makes a good experience?

...and how do we create them?

- What are the barriers to entry?
- What is the anchor to access the experience?
- Remove barriers, support, make connections through visualizations, language, and metaphor
7.N.6. Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially, and symbolically. 
[C, CN, PS, R, V]

- Explain, using concrete materials such as integer tiles and diagrams, that the sum of opposite integers is equal to zero.
- Illustrate, using a horizontal or vertical number line, the results of adding or subtracting negative and positive integers (e.g., a move in one direction followed by an equivalent move in the opposite direction results in no net change in position).
- Add two integers using concrete materials or pictorial representations, and record the process symbolically.
- Subtract two integers using concrete materials or pictorial representations, and record the process symbolically.
- Solve a problem involving the addition and subtraction of integers.
7.PR.2. Construct a table of values from a relation, graph the table of values, and analyze the graph to draw conclusions and solve problems.
[C, CN, R, V]

- Create a table of values for a relation by substituting values for the variable.
- Create a table of values using a relation, and graph the table of values (limited to discrete elements).
- Sketch the graph from a table of values created for a relation, and describe the patterns found in the graph to draw conclusions (e.g., graph the relationship between \( n \) and \( 2n + 3 \)).
- Describe the relationship shown on a graph using everyday language in spoken or written form to solve problems.
- Match a set of relations to a set of graphs.
- Match a set of graphs to a set of relations.
8.N.6. Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially, and symbolically.

[C, CN, ME, PS]

- Identify the operation(s) required to solve a problem involving positive fractions.
- Provide a context involving the multiplying of two positive fractions.
- Provide a context involving the dividing of two positive fractions.
- Express a positive mixed number as an improper fraction and a positive improper fraction as a mixed number.
- Model multiplication of a positive fraction by a whole number, concretely or pictorially, and record the process.
- Model multiplication of a positive fraction by a positive fraction, concretely or pictorially, and record the process.
- Model division of a positive fraction by a whole number, concretely or pictorially, and record the process.
- Generalize and apply rules for multiplying and dividing positive fractions, including mixed numbers.
- Solve a problem involving positive fractions, taking into consideration order of operations (limited to problems with positive solutions).
How do we formalize at the end?

- Build up to “a-ha” moments
- Create the need for formal language and formula authentically
- Teacher moves
THANK YOU!

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