Global Contributions to Mathematics

Intermediate Connections

You can find today’s presentation at the following link: https://bit.ly/GlobalMath78
Welcome!

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The only way to learn mathematics is to do mathematics.

Paul Halmos

Please ensure you have some paper and something to write with nearby!
MATH IS NOT NEUTRAL

Math and the math curriculum are NOT neutral. What is valued as mathematical, who is given credit for mathematical knowledge and how that mathematical knowledge is shared within the curriculum does not reflect global contributions to mathematics.
A Brief History of $\pi$

An exploration of its use around the world
In North America, the people of the First Nations would make baskets and drums of differing sizes, build with as little waste of materials as possible.

A drum frame would be made based on the size of the skin available. A basket would be sized according to its intended purpose. The specific purpose would determine the dimensions of the basket needed.

The late Mi’kmaw elder and quillbox maker, Dianne Toney, explained that to make a ring for a circular box top, she measured three times across the circle with her wood strips and added a thumb width. She declared it makes a perfect circle every time.

Try this with a 3” by 3” or 4” by 4” post it note and a piece of string…

The Daily Drum
Today’s drum features the “Woman in Prayer”. Prayer is given thanks on a daily basis for all creation—for fish, for food, for children, for Elders, for all the Creator has given us.
Have a Great Day!
King Solomon, ancient Israel (c 900 BCE)

“And he made a molten sea, ten cubits from the one brim to the other: it was round all about, and his height was five cubits: and a line of thirty cubits did compass it about...It was a handbreadth thick,...” (I Kings 7, 23 and 28)

In a list of specifications for the great temple of King Solomon, built around 950 BC and it seems that it gives \( \pi = 3 \).

But, if 10 cubits is entire diameter, 30 cubits is inside circumference (which would be circumference of the mold used) and 18 inches are in a cubit, then…this is where you can do the math … What is the approximate value of \( \pi \) used in ancient Israel?
## Some values of $\pi$ as known before 1600 AD

<table>
<thead>
<tr>
<th>Approx Date</th>
<th>Source/Mathematician</th>
<th>Method and Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>circa 1650 BC</td>
<td>Ahmes Papyrus, Egypt</td>
<td>Equate circular field to square field. $\pi \approx 3.16$</td>
</tr>
<tr>
<td>circa 1600 BC</td>
<td>Susa Tablet, Babylonia</td>
<td>Equating a regular hexagon to a circle. $\pi \approx 3.125$</td>
</tr>
<tr>
<td>circa 800 - 500 BC</td>
<td>Sulbasutras, India</td>
<td>Baudhayana gave a rule for area of equal squares and circles. $\pi \approx 3.09$</td>
</tr>
<tr>
<td>circa 250 BC</td>
<td>Archimedes, Greece</td>
<td>Compared perimeters of inscribed and circumscribed regular polygons. $\pi \approx 3.14$</td>
</tr>
<tr>
<td>circa 150 BC</td>
<td>Umasvati, India</td>
<td>Inscribing hexagon, polygons and using Pythagorean Theorem. $\pi \approx 3.16$</td>
</tr>
<tr>
<td>circa 260</td>
<td>Liu Hui, China</td>
<td>Inscribing polygons on various sizes up to 96 sides and Pythagorean Theorem. $\pi \approx 3.1416$</td>
</tr>
<tr>
<td>circa 480</td>
<td>Zu Chongzhi, China</td>
<td>As Liu Hui, but with polygons up to size 24,576 sides. $3.1415926 &lt; \pi &lt; 3.1415927$</td>
</tr>
<tr>
<td>circa 500</td>
<td>Aryabhata, India</td>
<td>Calculating perimeter of polygon of 384 sides. $\pi \approx 3.1416$</td>
</tr>
<tr>
<td>circa 1400</td>
<td>Madhava, India</td>
<td>Using an infinite series expansion. $\pi \approx 3.14159265359$ (correct to 11 decimal places)</td>
</tr>
<tr>
<td>circa 1429</td>
<td>Al-Kashi, Persia</td>
<td>Calculating the perimeter of a regular polygon with $3 \times 2^{28}$ sides. $\pi$ correct to 16 decimal places</td>
</tr>
<tr>
<td>circa 1579</td>
<td>Francois Viete, France</td>
<td>Calculating the perimeter of a regular polygon with 393216 sides. $\pi$ correct to 9 decimal places</td>
</tr>
</tbody>
</table>
Estimating $\pi$ Using Inscribed Polygons

A Brief Summary

The area of the inscribed polygon must be less than the area of the circle.

As the number of sides $n$ of the polygon grows, the area of the polygon gets closer to the area of the circle. Using the area of a circle formula, an estimate of $\pi$ can be obtained.
Liu Hui’s Method for finding $\pi$

Liu Hui noticed that if you know the perimeter of any regular polygon, it is relatively easy to calculate the perimeter of the regular polygon with twice as many sides. This can be repeated to get the perimeter of a polygon with four times as many sides. Then eight, sixteen, and so on.

He found a simple formula for finding the side length of a polygon with $2n$ sides if the side length of a polygon with $n$ sides was known. $S_n$ is perimeter of polygon with $n$ sides.

Let the radius be 1 to begin:

\[
\begin{align*}
  k_6 &= 1 \\
  k_{2n} &= \sqrt{2 + k_n} \\
  S_n &= \sqrt{2 - k_n}
\end{align*}
\]

Area of a polygon:

\[
P_n = n \left( \frac{1}{2} r S_n \right) = \frac{1}{2} n S_n
\]

Areas of circle and polygon are approximately equal as $n$ increases. We can solve for $\pi$.

\[
\begin{align*}
  \pi_n(1)^2 &\approx \frac{1}{2} n S_n \\
  \pi_6 &= 3\sqrt{2 - 1} = 3 \\
  \pi_{12} &= 6\sqrt{2 - \sqrt{2 + 1}} \approx 3.106 \\
  \pi_{24} &= 12\sqrt{2 - \sqrt{2 + \sqrt{2 + 1}}} \approx 3.133 \\
  \pi_{48} &= 24\sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + 1}}}} \approx 3.139
\end{align*}
\]
Area of a Circle - approximate using an Octagon

One of the major achievements of Egyptian Geometry is the approximations to the area of the circle.

The **Ahmes Papyrus** Problem 48 concerns measuring a circle using an octagon as an approximation.

It can be seen that the area of the octagon equals the area of the square (side length 9) minus the total area of the triangles cut off from the corners of the square.

\[
A = 9^2 - 4 \left(\frac{9}{2}\right) = 81 - 18 = 63
\]

How accurate is this model?
Area of a Circle and Conversions

One of the major achievements of Egyptian Geometry is the approximations to the area of the circle. The Egyptians developed a method that is not based on the recognition that the circumference is dependent on the diameter.

The **Ahmes Papyrus** Problem 50 reads as follows:

“A circular field has a diameter of 9 chet. What is its area?”

The solution reads: Subtract 1/9 of the diameter, namely 1 chet. The remainder is 8 chet. Multiply 8 by 8; it makes 64. Therefore it contains 64 cha-ta (square chet) of land.

Symbolically: \( A^E = \left[ d - \left( \frac{d}{9} \right) \right]^2 = \left( \frac{8d}{9} \right)^2 \) where \( d \) is diameter and \( A^E \) is area calculated the Egyptian way.

**What is the implicit Egyptian Calculation of \( \pi \)?**
The Egyptian Method for Arithmetic

A Case for Doubling and Halving
Egyptian Numeration

Three different notational systems developed over time

- Hieroglyphic - pictorial
  - Each character represented an easily recognizable object
  - Powers of 10
  - Operations: Addition and subtraction

- Hieratic - symbolic
  - Improved efficiencies over hieroglyphics

- Demotic - a later adaptation of the above

<table>
<thead>
<tr>
<th></th>
<th>Hieroglyphic</th>
<th>Hieratic</th>
<th>Demotic</th>
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</thead>
<tbody>
<tr>
<td>1</td>
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<td>1</td>
<td>1</td>
</tr>
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Crest of the Peacock, pg 84-87
Egyptian Arithmetic I

Multiplication and Division

● The method of ‘Duplation and Mediation’

● Requires
  ○ knowledge of addition
  ○ the two-times table
  ○ And that every integer can be written as a sum of powers of two

● Requires a form of ‘doubling and halving’

\[
\begin{array}{c|c|c}
13 \times 17 & \begin{array}{c|c|c}
\text{I} & \text{I} & \rightarrow 1 \\
\text{II} & \rightarrow 2 \\
\text{III} & \rightarrow 4 \\
\text{IV} & \rightarrow 8 \\
\end{array} \\
\text{I} \cap \text{II} & \text{III} & 1 + 4 + 8 = 13 \\
17 \times 68 & \text{I} \cap \text{II} & \rightarrow 17 \\
136 \times 68 & \text{I} \cap \text{II} & \rightarrow 68 \\
\text{I} \cap \text{II} & \text{III} & \rightarrow 136 \\
\end{array}
\]

\[17 + 68 + 136 = 221\]
You try!

Try solving the following using the Egyptian method #1.

11 x 15

11 groups of 15

1 group 15
2 groups 30
4 groups is 60
8 groups 120

8 + 2 + 1 = 11 groups
120 + 30 + 15 = 165
Egyptian Arithmetic I

Multiplication and Division

- The process of division was closely related to that of multiplication
- Egyptian thinking was “starting at 29 how many times should I add it to itself to get 696?”

<table>
<thead>
<tr>
<th>1</th>
<th>29</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>58</td>
</tr>
<tr>
<td>4</td>
<td>116</td>
</tr>
</tbody>
</table>

→ 8 232
→ 16 464

232 + 464 = 696
16 + 8 = 24
Egyptian Algorithm II

Multiplication and Division

- Still popular among rural communities in Russia, Ethiopia and Near East
- No multiplication tables, so the ability to double and halve numbers and distinguish from odd and even is all that is required
- Requires a form of ‘doubling and halving’
  - Expressing the multiplicand, 225 and then sum of integral powers of 2

<table>
<thead>
<tr>
<th></th>
<th>13</th>
<th>21</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>→</td>
<td></td>
</tr>
<tr>
<td>→</td>
<td>26</td>
<td>10</td>
</tr>
<tr>
<td>→</td>
<td>52</td>
<td>5</td>
</tr>
<tr>
<td>→</td>
<td>104</td>
<td>2</td>
</tr>
<tr>
<td>→</td>
<td>208</td>
<td>1</td>
</tr>
</tbody>
</table>

\[13 \times 21 = 13 + 52 + 208 = 273\]
Visual Representation of Egyptian Algorithm II

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>18</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>16</td>
<td>4</td>
</tr>
<tr>
<td>32</td>
<td>2</td>
</tr>
<tr>
<td>64</td>
<td>1</td>
</tr>
</tbody>
</table>

*(subtract 1 group = 8)*

*The area always stays the same!*
You try!

Try solving the following using the Egyptian method.

17 x 225

17 + 544 + 1088 + 2176 = 3825
Using Right Triangles
From Egyptian Rope Stretchers to Euclid
Egypt - Making a Right Triangle

As you go through various math classes, you will find 3-4-5 (and 5-12-13, 7-24-25, and others) triangles popping up in all sorts of problems.

Any multiples of 3-4-5 form similar triangles, which means that sides of 6-8-10 or 9-12-15 also make right angles.

Can you see why this is true?

The Egyptian surveyor could tie an extra knot in the middle of each space of his rope without changing the overall shape of the triangle. That would double the number of spaces on each side, turning 3-4-5 into 6-8-10.
Right Triangles

The earliest Chinese text on astronomy and mathematics, the Zhou Bi, is notable for a diagrammatic demonstration of the **gou gu** theorem.

\[ \text{gou}^2 + \text{gu}^2 = \text{xian}^2 \]

\( \text{gou} = \) smaller side  
\( \text{gu} = \) middle side  
\( \text{xian} = \) hypotenuse  

or \( a^2 + b^2 = c^2 \)

**The translation of the text:**

In terms of the diagram to the right, the larger square ABCD has side \( 3 + 4 = 7 \) and thus area 49. If, from this large square, four triangles (AHE, BEF, CFG, and DGH) making together two rectangles each of area of \( 3 \times 4 = 12 \), are removed, this leaves the smaller square HEFG.

\[ (3 + 4)^2 - 2(3 \times 4) = 3^2 + 4^2 = 5^2 \]
The Sulbasutras (c. 850 BC to 300 BC) were primarily instruction manuals for geometric constructions that had to conform to specified dimensions or areas. Any inaccuracies would make the consequent rituals and sacrifices ineffective.

For example: *to merge two equal or unequal squares to obtain a third square*

In modern notation,
- let ABCD and PQRS be the two squares to be combined
- let DX be equal to SR
- Draw a line to join A and X
- The square on AX is equal [in area] to the sum of the [area of the] squares of ABCD and PQRS

The original explanation then points out that $DX^2 + AD^2 = AX^2 = SR^2 + AD^2$, which shows the use of the Pythagorean Theorem.
The Plimpton Tablet and its connection to the side-length relationship for Right Angled Triangles

The tablet has four columns with 15 rows. The last column is the simplest to understand for it gives the row number and so contains 1, 2, 3, ..., 15. The remarkable fact is that in every row the square of the number c in column 3 minus the square of the number b in column 2 is a perfect square, say h.

Using modern day algebra: $c^2 - b^2 = h^2$

The table is a list of Pythagorean integer triples. The first entries are 119 and 169, corresponding to the Pythagorean triple $169^2 - 119^2 = 120^2$. 
The Plimpton Tablet and its connection to the side-length relationship for Right Angled Triangles

Generating the first three Pythagorean Triples from the Plimpton Tablet using Euclid’s equations.

<table>
<thead>
<tr>
<th>m</th>
<th>n</th>
<th>h = 2mn</th>
<th>b = m² - n²</th>
<th>d = m² + n²</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>64</td>
<td>27</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>32</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Slide this box to reveal
Euclid - Building any Pythagorean triple

For any s and t, s > t

\[ A = 2st \]

\[ B = s^2 - t^2 \]

\[ C = s^2 + t^2 \]

Let’s build a spreadsheet to generate and test these expressions
Euclid

Similar figures on the three sides

The Pythagorean theorem generalizes beyond the areas of squares on the three sides to any similar figures. This was known by Hippocrates of Chios in the 5th century BC, and was included by Euclid in his *Elements*:

If one erects similar figures (see Euclidean geometry) with corresponding sides on the sides of a right triangle, then the sum of the areas of the ones on the two smaller sides equals the area of the one on the larger side.

This extension assumes that the sides of the original triangle are the corresponding sides of the three congruent figures.
Thankfully, to the academic diligence of Joseph, and his groundbreaking book, *Crest of the Peacock: Non-European Roots of Mathematics*, we now know that the trajectory of mathematics reaching and being developed in Europe came through a rich reservoir of cultures and civilizations.

From the Ishango Bone of central Africa and the Inca *quipu* of South America to the dawn of modern mathematics, *The Crest of the Peacock* makes it clear that human beings everywhere have been capable of advanced and innovative mathematical thinking.
The History of Mathematics: An Introduction, covers the history behind the topics typically covered in an undergraduate math curriculum or in elementary schools or high schools. Elegantly written in David Burton's imitable prose, this classic text provides rich historical context to the mathematics that undergrad math and math education majors encounter every day.

Burton illuminates the people, stories, and social context behind mathematics' greatest historical advances while maintaining appropriate focus on the mathematical concepts themselves. Its wealth of information, mathematical and historical accuracy, is a valuable resource.
LIST OF IMPORTANT MATHEMATICIANS – TIMELINE

This is a chronological list of some of the most important mathematicians in history and their major achievements, as well as some very early achievements in mathematics for which individual contributions can not be acknowledged.

Where the mathematicians have individual pages in this website, these pages are linked; otherwise more information can usually be obtained from the general page relating to the particular period in history, or from the list of sources used. A more detailed and comprehensive mathematical chronology can be found at [http://www-groups.dcs.st-and.ac.uk/~history/Chronology/full.html](http://www-groups.dcs.st-and.ac.uk/~history/Chronology/full.html).
MacTutor History of Mathematics Archive

MacTutor is a free online resource containing biographies of more than 3000 mathematicians and over 2000 pages of essays and supporting materials.

MacTutor is constantly expanding and developing.

Recent changes to the archive (Up to MARCH 2022)

include 23 New Biographies (mainly of mathematicians who appear on our postage stamp page and did not previously have biographies) as well as 9 New Entries in the Additional Material category.

MacTutor was created and is maintained by Edmund Robertson and John O'Connor of the School of Mathematics and Statistics at the University of St Andrews, and is hosted by the University. Their contributions to the history of mathematics have been recognised by the Comenius medal of the Hungarian Comenius Society in 2012 and the Hirst Prize of the London Mathematical Society in 2015.
HISTORY OF MATHEMATICS PROJECT

A virtual interactive exhibit being developed for the National Museum of Mathematics in New York City
Any Questions?