

Please work on the following question:

Many numbers can be expressed as the difference of two perfect squares. For example,

$$20 = 6^2 - 4^2$$

$$21 = 5^2 - 2^2$$

$$36 = 6^2 - 0^2$$

How many of the numbers from 1 to 30 can you express as the difference of two perfect squares?

Integrating Problem Solving in Grades 9 and 10

CEMC: Bringing Teachers Together Virtually
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**Integrating
Problem
Solving in
Grades 9 and 10**

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Back to the Question:

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How many of the numbers from 1 to 30 can you express as the difference of two perfect squares?

What was your process?

Hopefully you found something like this:

1: $1^2 - 0^2$ is $1 - 0 = 1$

2: does not work

3: $2^2 - 1^2$ is $4 - 1 = 3$

4: $2^2 - 0^2$ is $4 - 0 = 4$

5: $3^2 - 2^2$ is $9 - 4 = 5$

6: does not work

7: $4^2 - 3^2$ is $16 - 9 = 7$

8: $3^2 - 1^2$ is $9 - 1 = 8$

9: $5^2 - 4^2$ is $25 - 16 = 9$

10: does not work

11: $6^2 - 5^2$ is $36 - 25 = 11$

12: $4^2 - 2^2$ is $16 - 4 = 12$

13: $7^2 - 6^2$ is $49 - 36 = 13$

14: does not work

15: $8^2 - 7^2$ is $64 - 49 = 15$

16: $5^2 - 3^2$ is $25 - 9 = 16$

17: $9^2 - 8^2$ is $81 - 64 = 17$

18: does not work

19: $10^2 - 9^2$ is $100 - 81 = 19$

20: $6^2 - 4^2$ is $36 - 16 = 20$

21: $11^2 - 10^2$ is $121 - 100 = 21$

22: does not work

23: $12^2 - 11^2$ is $144 - 121 = 23$

24: $7^2 - 5^2$ is $49 - 25 = 24$

25: $13^2 - 12^2$ is $169 - 144 = 25$

26: does not work

27: $14^2 - 13^2$ is $196 - 169 = 27$

28: $8^2 - 6^2$ is $64 - 36 = 28$

29: $15^2 - 14^2$ is $225 - 196 = 29$

30: does not work

What kind of extensions could you do with this question?

Given an odd number, what difference of squares generates it?

What numbers do not work?

Difference of squares for numbers differing by 2? 3? 4?

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Organize, plan and write the Halton Math Contest - team competition

- spend up to two months looking for good problems

Teach IB Higher and Standard Level, Ontario Mathematics Curriculum and Computer Science

The mathematical processes that support effective learning in mathematics are as follows:

- problem solving
- reasoning and proving
- reflecting
- connecting
- communicating
- representing
- selecting tools and strategies

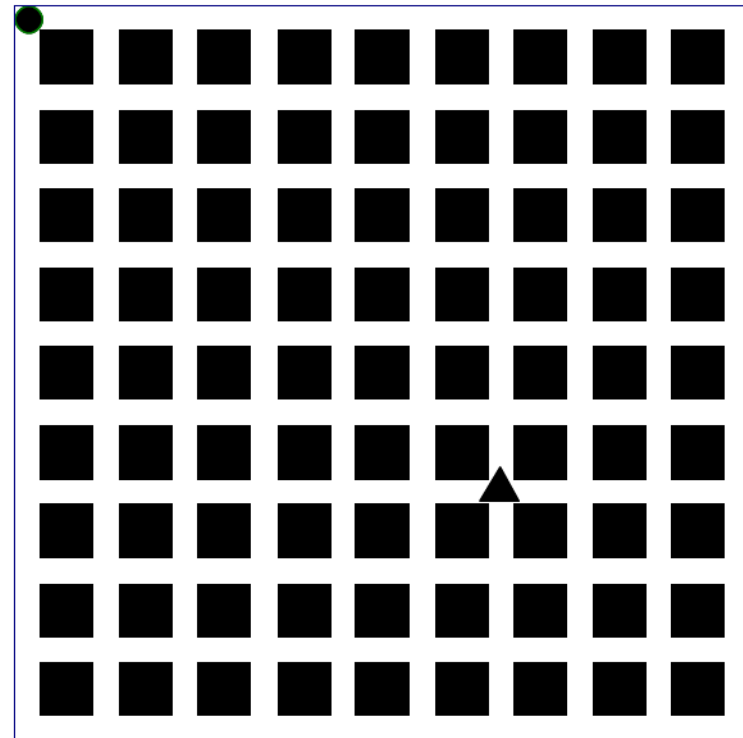
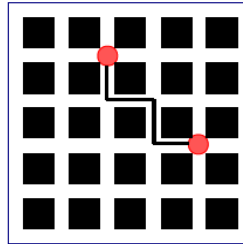
Taxicab Geometry

To get around in a city, you can only travel along the streets using a taxicab as in the diagram to the right.

The large diagram below represents the map of the city you live in.

You live at ● and work at ▲ .

There is a shortest path to travel from where you live to where you work. How many different routes are there that have a length equal to the shortest path?

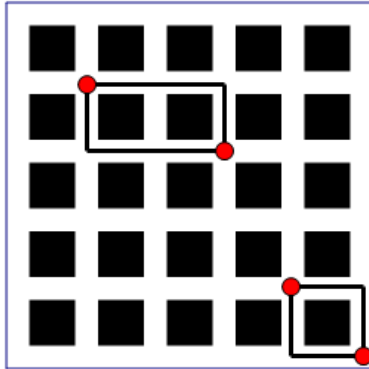


Taxicab Geometry Ext #1

In Taxicab Geometry, biangles exist.

A biangle has two equal sides between two points.

Imagine triangles, just with only two equal sides.



The diagram to the left shows a biangle of side length 2 and a biangle of side length 3.

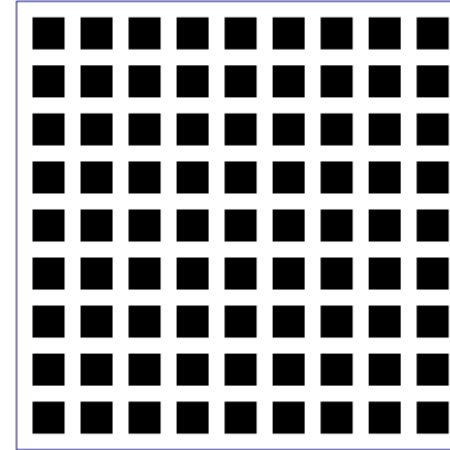
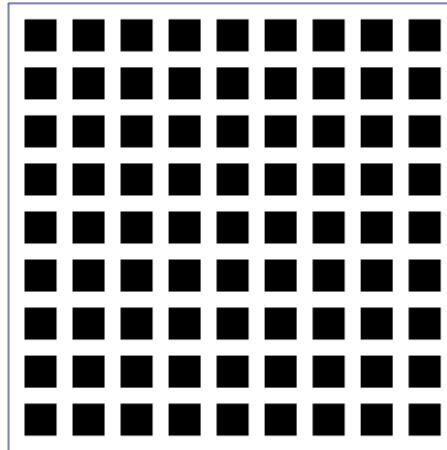
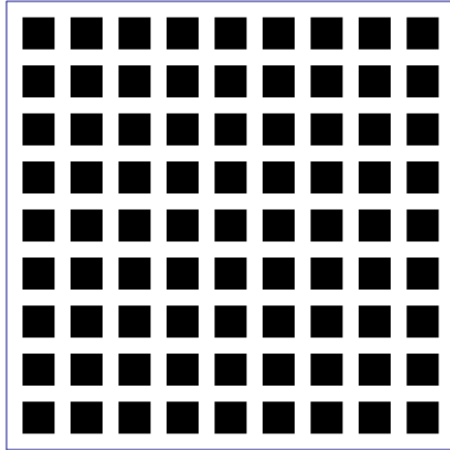
What is the area (number of city blocks) of each?

Are there any other biangles of side length 2 and 3 you can find?

What are their areas?

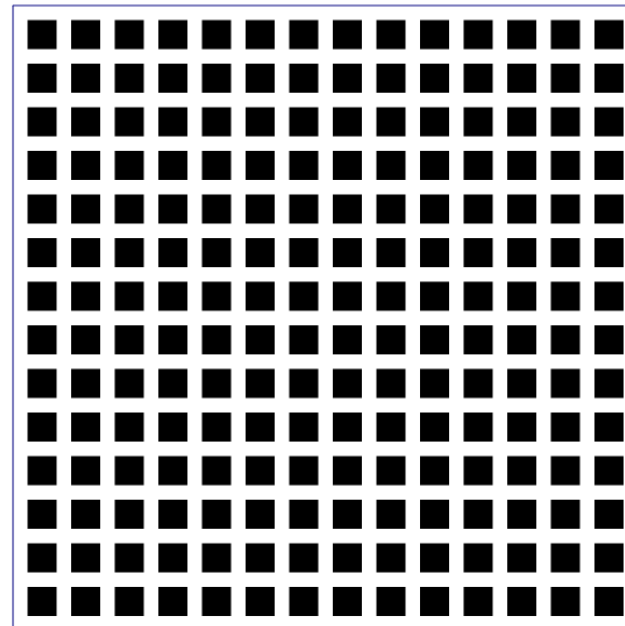
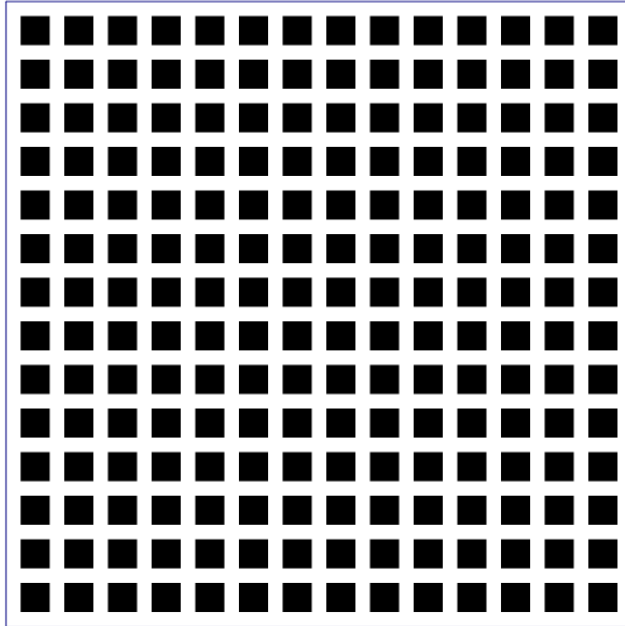
Taxicab Geometry Ext #2

Can you find biangles of length 3 to 8?
What are their possible areas?



Taxicab Geometry Ext #3

Examine triangles, squares, etc... in Taxicab Geometry
Include areas in your exploration.

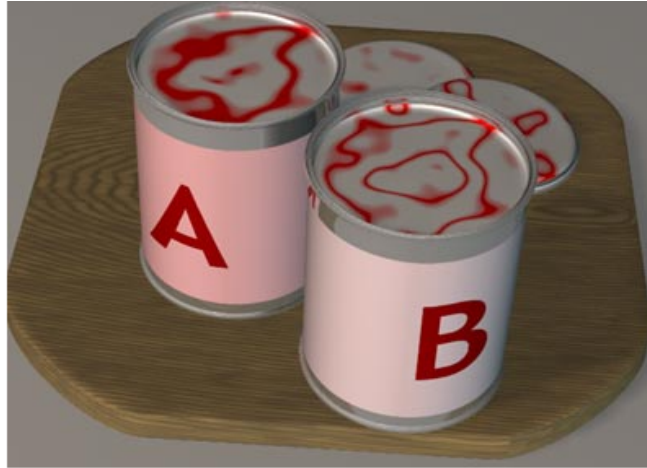


Ratio Fun

A decorator can buy pink paint from two manufacturers.

- Paint A is made up from red and white paint in the ratio $1 : 3$
- Paint B is made up from red and white paint in the ratio $1 : 7$

The decorator can mix the paints to produce different shades of pink.



If Paint A and Paint B come in the same size cans, what is the least number the decorator would need of each type in order to produce pink paint containing red and white in the following ratios?

$1 : 4$
 $1 : 5$
 $1 : 6$

Mean-Median-Mode

There are several sets of five positive whole numbers with the following properties:

- Mean = 4
- Median = 3
- Mode = 3

Can you find **all** the different sets of five positive whole numbers that satisfy these conditions?

Can you explain how you know you've found them all?

Extension

Here's an interesting set of five numbers:

2, 5, 5, 6, 7

The mean, mode, median and range are all 5.

Can you find other sets of five positive whole numbers where:

$$\text{Mean} = \text{Median} = \text{Mode} = \text{Range}$$

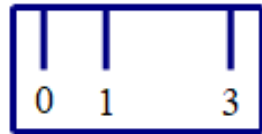
Can you find sets of five positive whole numbers that satisfy the following properties?

- A. Mode < Median < Mean
- B. Mode < Mean < Median
- C. Mean < Mode < Median
- D. Mean < Median < Mode
- E. Median < Mode < Mean
- F. Median < Mean < Mode

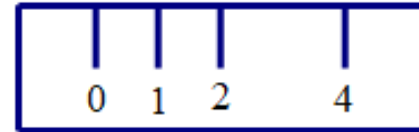
Perfect Golomb Rulers

- used to measure all lengths from 1 unit to n units
- no two measurements are represented twice

Why is this ruler perfect?



Why is this ruler not perfect?



The next perfect Golomb ruler is of length 6.
Can you find it?

For more Golomb fun, here's a website!

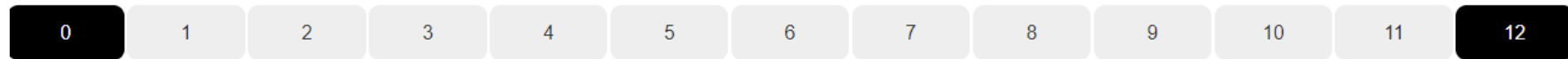
Ruler Investigation Tool

Your goal is to construct a ruler that can measure every integer value from 1 inch to the length of the ruler by placing as few marks on the ruler as possible. The measurements must be done without moving the ruler, so you must be able to measure any integer length between marks that exist on your ruler.

You will start with a 12 inch long ruler with no marks. At first, the only length this ruler can accurately measure is 12 inches.

Example: If you place at the 3 inch mark, then you will be able to measure 3 inches (0 to 3) AND 9 inches (3 to 12).

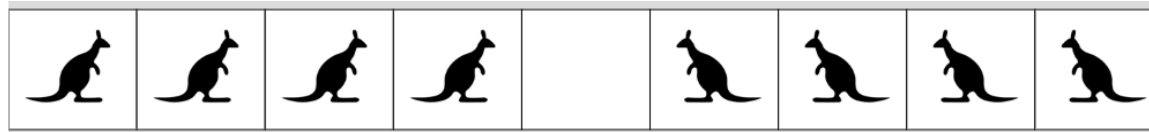
Add marks to the ruler one at a time and the tool will tell you which measurements you are able to make.

<http://gadgets.mathplusacademy.com/ruler/ruler.html>



Kanagaroos Must Pass!!



Two groups of kangaroos meet on a narrow mountain path.

The challenge is to swap places using the following rules:

- 1) Only one kangaroo can move at a time. A move is a hop or a jump.
- 2) Kangaroos can hop one space forward, never backward, to an open space.
- 3) A kangaroo can jump over one **on-coming kangaroo** at a time and must land on an open space.

Can you successfully swap their positions? <https://kangaroos.jasonvan.repl.co/>

Extensions

What is the least number of moves required to swap the kangaroos?

What if there are five kangaroos on each side?

Ten kangaroos on each side?

n kangaroos on each side?

Puzzles:

The following puzzle is called a Slitherlink and are from krazydad.com.

A slitherlink consists of a grid of dots, with some clue cells containing numbers.

You connect horizontally or vertically adjacent dots to form a meandering path that forms a single loop or "slitherlink".

The loop must not have branches and must not cross itself.

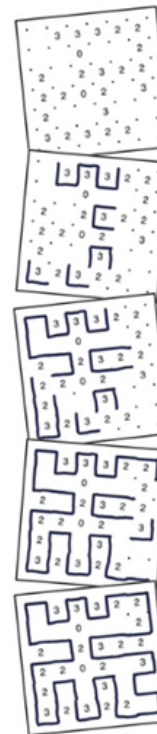
The clue numbers indicate how many lines surround the cell.

Empty cells may be surrounded by any number of lines (0 to 3).

There is only one unique solution and you should be able to find it without guessing.

You may find it helpful to make X's between dots that cannot be connected.

To the right is an example of a slitherlink in various stages of solving.



<https://tinyurl.com/yk93knp7>

Logic Problems:

In a new television series there are 1000 entrants.

All 1000 entrants were picked from a hat and the order of picking was kept and each entrant thus given a number.

The first person picked, is entrant #1, the second person is entrant #2, and so on until the last person picked who is entrant #1000.

There is a total purse of \$10 000 000 and the entrants have to decide how to split up these winnings.

Every day the entrants will vote to either vote off the lowest ranking entrant (higher the number, the lower the rank), or split the winnings among those entrants still in the game.

If 50% or more of the entrants vote to split the winnings, the winnings get split.

Otherwise, the lowest ranking entrant is removed from the game.

Each day the game continues, with the entrants voting and either splitting the winnings (thus ending the game) or the lowest ranking entrant being removed.

All the contestants are extremely greedy, heartless, and perfectly rational.

On which day will the winnings be split?

Thank you!

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