University of Waterloo
Faculty of Mathematics

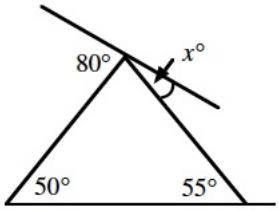
Centre for Education in
Mathematics and Computing

Pascal and Cayley Contest Preparation

Problem Set 1

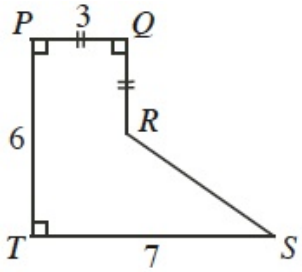
1. A bag contains 5 red, 6 green, 7 yellow, and 8 blue jelly beans. A jelly bean is selected at random. What is the probability that it is blue?
   (A) $\frac{5}{26}$ (B) $\frac{3}{13}$ (C) $\frac{7}{26}$ (D) $\frac{4}{13}$ (E) $\frac{6}{13}$

2. The value of x is



   (A) 25 (B) 30 (C) 50 (D) 55 (E) 20

3. In the diagram, what is the perimeter of polygon $PQRST$?

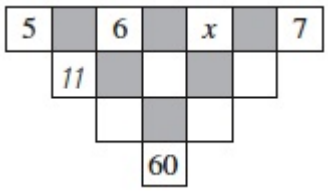


   (A) 24 (B) 23 (C) 25 (D) 26 (E) 27

   Hint: Drop a perpendicular line from the point R to the line ST and use the lengths you know and the Pythagorean Theorem to find the length RS.

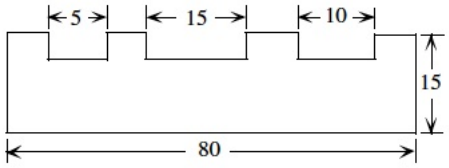
4. The vertices of a triangle have coordinates (1,1), (7,1) and (5,3). What is the area of this triangle?
   (A) 12 (B) 8 (C) 6 (D) 7 (E) 9

5. The number in an unshaded square is obtained by adding the numbers connected to it from the row above. The value of x must be



(A) 4 (B) 6 (C) 9 (D) 15 (E) 10

6. Three small rectangles, of the same depth, are cut from a rectangular sheet of metal. The area of the remaining piece is 990. What is the depth of each cut?



(A) 8 (B) 7 (C) 6 (D) 5 (E) 4

Hint: Calculate all areas seperately using a variable for the depth and them sum them to 990.

7. If the surface area of a cube is 54, what is its volume?

(A) 36 (B) 9 (C) $\frac{81\sqrt{3}}{8}$ (D) 27 (E) $162\sqrt{6}$

8. The numbers $6, 14, x, 17, 9, y, 10$ have a mean of 13. What is the value of $x + y$?
(A) 20 (B) 21 (C) 23 (D) 25 (E) 35

9. In the magic square, the sum of the three numbers in any row, column or diagonal are the same.

$2x$	3	2
		-3
0	x	

The sum of the three numbers in any row is
(A) 0 (B) 1 (C) 3 (D) 7 (E) 9

Hint: Keep in mind the question is asking for the sum of the numbers in any row/column, not the value of x. There are different techniques to solve this problem. For example, you could equate the 1st row and the 3rd row to figure out what goes in the bottom-right square and equate the 3rd column and 3rd row to find x and use that to find the sum. Another technique is equating the first 2 values from the 3rd column and the first 2 values in the 3rd row to find the value of x and use that to find the sum. Also, you could do trial and error,

using the answers that are given.

10. A light flashes every 6 minutes and a bell rings every 8 minutes. If the light flashes as the bell is ringing, then the smallest number of minutes which must elapse before this occurs is
(A) 14 (B) 42 (C) 48 (D) 72 (E) 24

11. An *ascending* integer occurs when each digit is greater than the digit on its left. An example is 2478. The number of ascending integers between 4000 and 5000 is
(A) 7 (B) 8 (C) 9 (D) 10 (E) 11

12. How many integers between 2 and 50 can be written of the form x^y, where x and y are positive integers and $y \neq 1$?
(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

Hint: Carefully read the question. Keep in mind that the question asks for the number of integers, not the number of possibilities of x^y. The wording is a bit tricky and can lead to a different answer.

13. When 54 is subtracted from a two-digit number, the result is a number having the same two digits reversed in order. Determine all two-digit numbers satisfying this property.

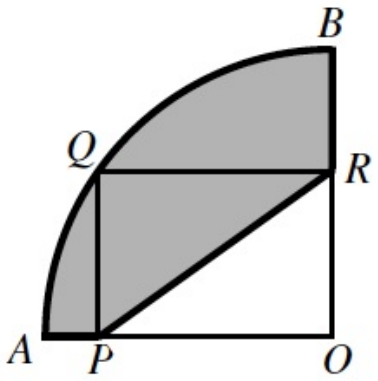
Hint: Read the question carefully. It asks for all two-digit numbers which eliminates one of the possible values. To solve this problem algebraically, let the 2-digit number be $xy = 10x + y$. Since the result will have the number in reversed order, the result would be $yx = 10y + x$. You can use these two expressions to make an equation that you can solve.

Eliminate some distractors:

1. A chocolate drink is 6% pure chocolate, by volume. If 10 litres of pure milk are added to 50 litres of this drink, the percent of chocolate in the new drink is
(A) 5 (B) 16 (C) 10 (D) 3 (E) 26

Hint: Right away we can eliminate some obvious wrong answers. For instance, we can eliminate any number greater than 6 since we are adding pure milk with no percentage of chocolate to a chocolate drink that has 6% chocolate. This means that the percentage of chocolate in the drink will decrease. So, we can see that B, C and E are not correct. Now there is a 50-50 chance on getting the answer correct even if you did not know how to solve the problem.

2. In the diagram, AOB is a quarter circle of radius 10 and $PQRO$ is a rectangle of perimeter 26. The perimeter of the shaded region is



(A) $7+5\pi$ (B) $13+5\pi$ (C) $17+5\pi$ (D) $7+25\pi$ (E) $17+25\pi$

Hint: By doing some work, we can eliminate some possibilities. Since the radius of the circle is 10, we can find the circumference to be $2\pi r = 20\pi$. So if we want to find the arc length of a quarter of this circle (because the question asks for the perimeter) then the arc length would be $\frac{20\pi}{4} = 5\pi$. So we can conclude that a number plus 25π is highly unlikely and we can eliminate both D and E.

Try plugging in values!

1. The smallest positive integer x for which the sum $x + 2x + 3x + 4x + ... + 100x$ is a perfect square is
(A) 50 (B) 100 (C) 101 (D) 202 (E) 5050

Hint: Try plugging in each possible answer given. Since you want the smallest positive integer, start with A and work your way up.

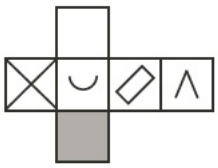
2. The products mn and rs are both 24. Both m and r have values between 1 and 24. If m is less than r, then s is

(A) less than 24 and less than n (B) greater than 24 and less than n
(C) equal to n (D) greater than 24 and greater than n
(E) less than 24 and greater than n

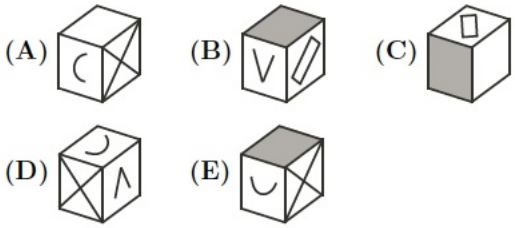
Hint: Try different values for m and r where the condition holds. Then find the corresponding values of s and n and see which answer fits.

Try constructing the object!

1. The diagram shows a piece of cardboard that can be folded to make a cube.
   The cardboard has designs on one side only.



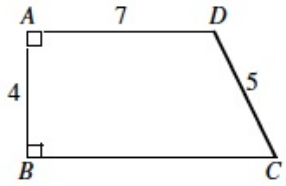
Which one of the following cubes can be made from this cardboard?



Hint: If you are having trouble visualizing what the cube will look like from the net given, try using scrap paper to create the object and use that to figure out which answer is correct.

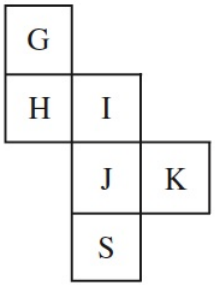
Problem Set 2

1. In the diagram, $AD < BC$. What is the perimeter of $ABCD$?



(A) 23 (B) 26 (C) 27 (D) 28 (E) 30

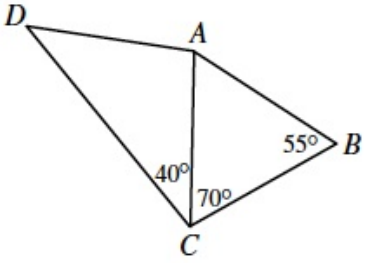
2. If the figure is folded to make a cube, what letter is opposite G?



(A) S (B) H (C) I (D) J (E) K

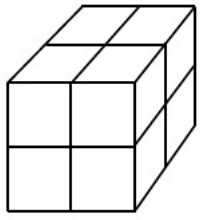
3. A store had a sale on T-shirts. For every two T-shirts purchased at the regular price, a third T-shirt was bought for \$1.00. Twelve T-shirts were bought for \$120.00. What was the regular price for one T-shirt?
(A) \$10.00 (B) \$13.50 (C) \$14.00 (D) \$14.50 (E) \$15.00

4. In the diagram, $DA = CB$. What is the measure of $\angle DAC$?



(A) 70° (B) 100° (C) 95° (D) 125° (E) 110°

5. Eight unit cubes are used to form a large 2 by 2 by 2 cube. The six faces of this larger cube are then painted red. When the paint is dry, the larger cube is taken apart. What fraction of the total surface area of the unit cubes is red?



(A) $\frac{1}{6}$ (B) $\frac{2}{6}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$ (E) $\frac{1}{3}$

6. When 14 is divided by 5, the remainder is 4. When 14 is divided by the positive integer n, the remainder is 2. For how many different values of n is this possible?
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

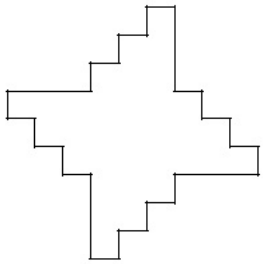
7. A gardener has a push mower and a riding mower. It takes her five hours to cut the entire lawn the the push mower but only 70 minutes with the riding mower. After 90% of the lawn was cut using the riding mower, the remainder was cut using the push mower. How many minutes did it take to cut the lawn?

(A) 120 (B) 75 (C) 70 (D) 277 (E) 93

8. How many integers are between $\sqrt{40}$ and $\sqrt{400}$?

(A) 12 (B) 13 (C) 14 (D) 15 (E) 16

9. In the diagram, adjacent edges are at right angles. The four longer edges are equal in length, and all of the shorter edges are also equal in length. The area of the shape is 528. What is the perimeter?



(A) 132 (B) 264 (C) 92 (D) 72 (E) 144

10. The digits 1, 2, 3, 4, 5 and 6 are each used once to compose a six digit number $abcdef$, such that the three digit number abc is divisible by 4, bcd is divisible by 5, cde is divisible by 3, and def is divisible by 11. The digit a is
(A) 1 (B) 2 (C) 3 (D) 4 (E) 6

11. If n is a positive integer and $n(n+1)$ is divided by 3, the remainder can be
(A) 0 only (B) 2 only (C) 0 or 1 only (D) 0 or 2 only (E) 0, 1, or 2

12. Suppose that a, b and c are three numbers with

$$\begin{aligned} a+b &= 3 \\ ac+b &= 18 \\ bc+a &= 6 \end{aligned}$$

The value of c is
(A) 2 (B) 11 (C) 3 (D) 6 (E) 7

Extra Problems

1. The numbers 49, 29, 9, 40, 22, 15, 53, 33, 13, 47 are grouped in pairs so that the sum of each pair is the same. Which number is paired with 15?
   (A) 33 (B) 40 (C) 47 (D) 49 (E) 53

2. When the expression $2005^2 + 2005^0 + 2005^0 + 2005^5$ is evaluated, the final two digits are
   (A) 52 (B) 25 (C) 20 (D) 50 (E) 05

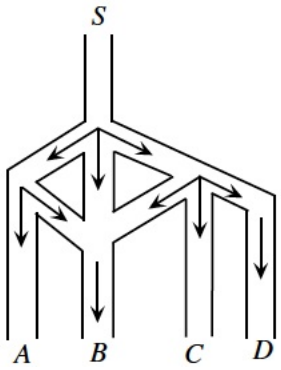
3. The number of positive integers that are less than 500 and that are not divisible by 2 or 3 is
   (A) 168 (B) 167 (C) 166 (D) 165 (E) 83

4. If $4 \leq x \leq 12$ and $6 \leq y \leq 10$, and $S = x - y$, what is the largest possible interval for S?

5. The five-digit number $9T67U$, where T and U are digits, is divisible by 36. Determine all possible values for T and U.

6. Using only odd digits, all possible three-digit numbers are formed. Determine the sum of all such numbers.

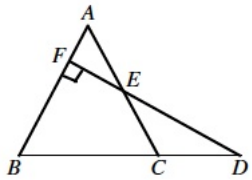
7. Harry the Hamster is put in a maze, and he starts at point S. The paths are such that Harry can move forward only in the direction of the arrows. At any junction, he is equally likely to choose any of the forward paths. What is the probability that Harry ends up at B?



(A) $\frac{2}{3}$ (B) $\frac{13}{18}$ (C) $\frac{11}{18}$ (D) $\frac{1}{3}$ (E) $\frac{1}{4}$

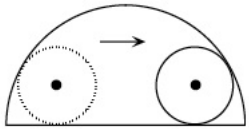
8. If $90! = (90)(89)(88)\cdots(2)(1)$, then the exponent of the highest power of 2 that will divide 90! is
(A) 86 (B) 45 (C) 90 (D) 75 (E) 85

9. In the diagram, $\triangle ABC$ is equilateral, $BC = 2CD$, $AF = 6$, and DEF is perpendicular to AB. What is the area of quadrilateral $FBCE$?



(A) $144\sqrt{3}$ (B) $138\sqrt{3}$ (C) $126\sqrt{3}$ (D) $108\sqrt{3}$ (E) $66\sqrt{3}$

10. A wheel of radius 8 rolls along the diameter of a semicircle of radius 25 until it bumps into this semicircle. What is the length of the portion of the diameter of the semicircle that cannot be touched by the wheel?



(A) 8 (B) 12 (C) 15 (D) 17 (E) 20

Answers

Problem Set 1

1. D
2. A
3. A
4. C
5. E
6. B
7. D
8. E
9. C
10. E
11. D
12. D
13. 3

Eliminate Some Distractors

1. A
2. C

Try plugging in values

1. D
2. A

Try Constructing the object

1. B

Problem Set 2

1. B
2. D
3. D
4. B
5. C
6. D
7. E
8. B
9. E
10. C
11. D
12. E

Extra Problems

1. C
2. A
3. B
4. [-6, 6]
5. $T = 3, U = 2$ and $T = 8, U = 6$
6. 69375
7. C
8. A
9. C
10. E