University of Waterloo
Faculty of Mathematics

Centre for Education in
Mathematics and Computing

Intermediate Math Circles
February 3, 2010
Contest Preparation I

The Pascal and Cayley contests are the multiple choice Grade 9 and Grade 10 contests.

Each contest has 25 multiple choice problems. The problems are meant to gradually increase in difficulty, so problem 1 should be the easiest and problem 25 the hardest.

The problems are split up into three sections. Section A has ten questions each worth 5 points, section B has ten questions each worth 6 points and section C has five questions each worth 8 points. In addition, any question left unanswered is worth 2 points up to a maximum of 20 points. This is important to consider as it means guessing is not always rewarded.

Most of the following problems have been taken from past Pascal and Cayley contests.

Problem Set 1

1. What is the value of $1 + 2 \times 9 - \sqrt{36}$?
   (A) 7 (B) 11 (C) 8 (D) 13 (E) 4

2. The areas of three squares are 16, 49 and 169. What is the average (mean) of their side lengths?
   (A) 8 (B) 12 (C) 24 (D) 39 (E) 32

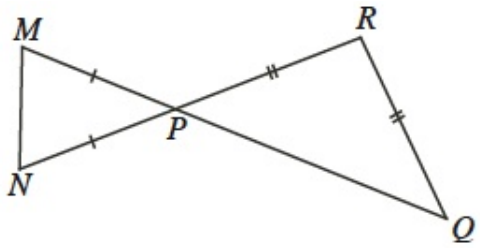
3. In the subtraction shown, M and N each represent a single digit. What is the value of $M + N$?

$$\begin{array}{r} \boxed{M}\boxed{4} \\ -\ \boxed{3}\boxed{N} \\ \hline \boxed{1}\boxed{6} \end{array}$$

   (A) 14 (B) 12 (C) 15 (D) 13 (E) 11

4. A class of 30 students recently wrote a test. If 20 students scored 80, 8 students scored 90, and 2 students scored 100, then the class average on this test was
   (A) 90 (B) 84 (C) 82 (D) 86 (E) 88

5. In the diagram, if $\angle PQR = 48°$, what is the measure of $\angle PMN$?



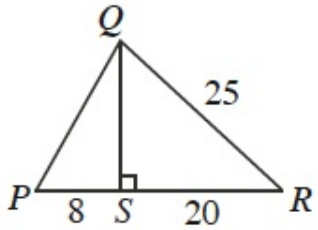
(A) 60° (B) 42° (C) 48° (D) 66° (E) 84°

6. For how many different values of k is the 4-digit number $7k52$ divisible by 12?
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

7. The point $(0, 0)$ is reflected in the vertical line $x = 1$. When its image is then reflected in the line $y = 2$, the resulting point is
(A) $(0, 0)$ (B) $(2, 0)$ (C) $(4, 4)$ (D) $(2, 2)$ (E) $(2, 4)$

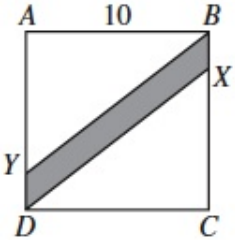
8. At Webster High School, the ratio of males to females writing the Pascal Contest is 3 : 7. If there are 21 males writing the Contest, what is the *total* number of students writing?
(A) 30 (B) 25 (C) 49 (D) 70 (E) 79

9. In the diagram, what is the perimeter of $\triangle PQR$?



(A) 63 (B) 60 (C) 55 (D) 85 (E) 70

10. In the diagram, $ABCD$ is a square with a side length of 10. If $AY = CX = 8$, the area of the shaded region is



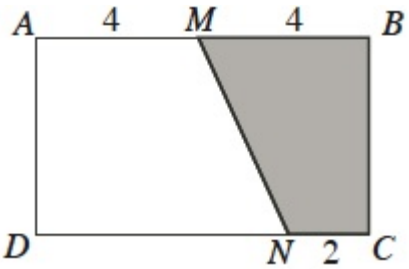
(A) 16 (B) 20 (C) 40 (D) 48 (E) 24

11. At Springfield University, there are 10000 students, and there are as many male students as female students. Each student is enrolled either in the Arts program or Science program (but not both); 60% of the students are in the Arts program. Also, 40% of the Science students are male. To the nearest percent, what percentage of the Arts students are female?
(A) 50% (B) 52% (C) 26% (D) 65% (E) 43%

12. In a right-angled triangle, the sum of the squares of the three side lengths is 1800. The length of the hypotenuse is
(A) $\sqrt{1800}$ (B) $\frac{1}{2}\sqrt{1800}$ (C) 90 (D) 30 (E) 45

Narrowing down choices:

1. In the diagram, the area of rectangle $ABCD$ is 40. The area of $MBCN$ is

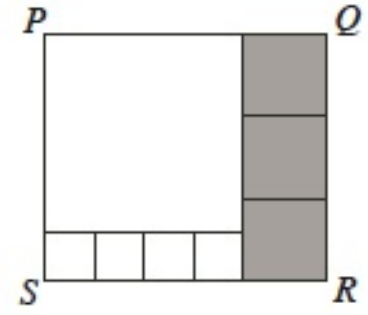


(A) 15 (B) 10 (C) 30 (D) 12 (E) 16

We can see in the diagram that M is the midpoint of AB. We can also see that NC is one quarter of the length of DC. As a result, we can tell that the area of the shaded region is more than one quarter, but less than one half of the area of the rectangle.

This means the area of the shaded region is more than 10 but less than 20. Right away we can discount (B) 10 and (C) 30 as answers.

2. Rectangle $PQRS$ is divided into eight squares, as shown. The side length of each shaded square is 10. What is the length of the side of the largest squares?



(A) 18 (B) 24 (C) 16 (D) 23 (E) 25

From the diagram, we can see that the side of the large square is longer than two side lengths of the shaded squares. This means the side length of the large square must be more than $2 \times 10 = 20$. We can discount (A) 18 and (C) 16 as answers.

Plug in the given values!

1. When $x = 9$, which of the following has the largest value?
   (A) $\sqrt{x}$ (B) $\frac{x}{2}$ (C) $x - 5$ (D) $\frac{40}{x}$ (E) $\frac{x^2}{20}$

   Instead of considering each of the answers with the variable x still in it, simply plug in $x = 9$ and calculate the value of each answer.

2. If $a = 7$ and $b = 13$, the number of even positive integers less than ab is
   (A) $\frac{ab-1}{2}$ (B) $\frac{ab}{2}$ (C) $ab - 1$ (D) $\frac{a+b}{4}$ (E) $(a-1)(b-1)$

   Once again, substitute $a = 7$ and $b = 13$ into the answers and calculate the value of each answer.

Try writing out a few cases!

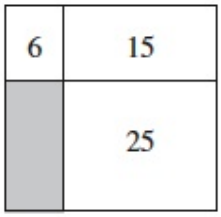
3. When three consecutive positive integers are multiplied together, the answer is always
   (A) odd (B) a multiple of 6 (C) a multiple of 12 (D) a multiple of 4
   (E) a multiple of 5

   Three possible consecutive positive integers are 1, 2 and 3. $1 \times 2 \times 3 = 6$ and 6 is not odd, is not a multiple of 12, is not a multiple of 4 and is not a multiple of 5. Therefore, the only answer that fits is (B) a multiple of 6.

Problem Set B

1. The odometer of a car reads 2722 km. The least number of kilometers that must be travelled before the odometer will again show a number in which three digits are the same is between
   (A) 0 and 50 (B) 50 and 100 (C) 100 and 500 (D) 500 and 1000
   (E) 1000 and 5000

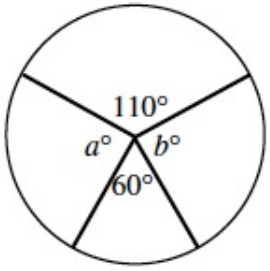
2. A rectangle is divided into four smaller rectangles. The areas of three of these rectangles are 6, 15 and 25, as shown. The area of the shaded rectangle is



(A) 7 (B) 15 (C) 12 (D) 16 (E) 10

3. If $a * b$ is defined as $(a + 1)(b - 1)$, what is the value of $0 * 0$?
(A) 0 (B) -1 (C) 1 (D) 2 (E) -2

4. In the diagram, $a + b$ equals



(A) 10 (B) 85 (C) 110 (D) 170 (E) 190

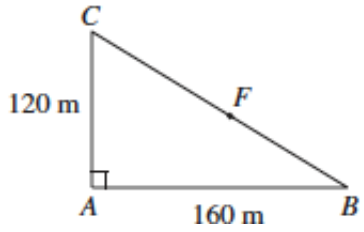
5. Starting at 777 and counting backwards by 7s, a student counts 777, 770, 763, etc. A number that will be counted is
(A) 45 (B) 44 (C) 43 (D) 42 (E) 41

6. Chris and Pat are planning a meal together. Chris spends \$8.43 at one grocery store and \$13.37 at another. At a third store, Pat bought \$2.46 worth of groceries. If the cost of the dinner is to be split evenly, how much does Pat owe Chris?
(A) \$10.90 (B) \$8.46 (C) \$19.34 (D) \$2.48 (E) \$9.67

7. The sum of five consecutive integers is 75. The sum of the largest and smallest of these five integers is
(A) 17 (B) 65 (C) 13 (D) 30 (E) 34

8. After having played three basketball games, Megan had scored an average of 18 points per game. After her fourth game, her scoring average dropped to 17 points per game. How many points did Megan score in her fourth game?
(A) 18 (B) 17 (C) 16 (D) 15 (E) 14

9. In the diagram, ABC represents a triangular jogging path. Jack jogs along the path from A to B to F. Jill jogs from A to C to F. Each jogs the same distance. The distance from F to B in metres is

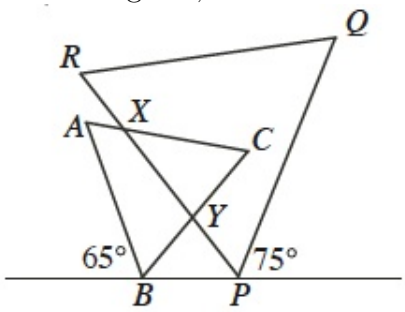


(A) 40 (B) 120 (C) 100 (D) 80 (E) 200

10. How many numbers n between 10 and 200 are such that n is prime and $n-1$ is a perfect square?
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

11. When the product $(5^3)(7^{52})$ is expanded, the units digit is
(A) 5 (B) 3 (C) 9 (D) 7 (E) 0

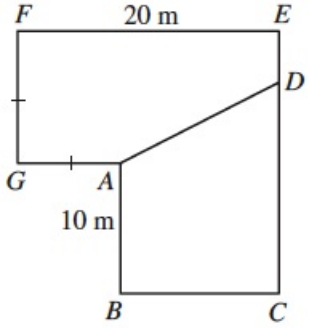
12. In the diagram, if $\triangle ABC$ and $\triangle PQR$ are equilateral, then $\angle CXY$ equals



(A) 30° (B) 35° (C) 40° (D) 45° (E) 50°

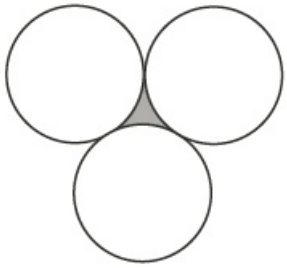
13. On an island there are two types of inhabitants: Heros who always tell the truth and Villains who always lie. Four inhabitants are seated around a table. When each is asked "Are you a Hero or a Villain?", all four reply "Hero". When asked "Is the person on your right a Hero or Villain?", all four reply "Villain". How many Heros are present?
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

14. In the diagram, $ABCDEFG$ is a room having square corners, with $EF = 20$m, $AB = 10$m, and $AG = GF$. The total area of the room is $280m^2$. A wall is built from A to D creating two rooms of equal area. What is the distance, in metres, from C to D?



(A) 15 (B) $\frac{50}{3}$ (C) 12 (D) 13 (E) $\frac{40}{3}$

15. In the diagram, each of the three identical circles touch the other two. The circumference of each circle is 36. What is the perimeter of the shaded region?



(A) 18 (B) 6 (C) 36 (D) 12 (E) 24

Problem Set C

1. If $x = 3$, which of the following expressions is an even number?
(A) $9x$ (B) x^3 (C) $2(x^2 + 9)$ (D) $2x^2 + 9$ (E) $3x^2$

2. Carly takes three steps to walk the same distance as Jim walks in four steps. Each of Carly's steps covers 0.5 metres. How many metres does Jim travel in 24 steps?
(A) 16 (B) 9 (C) 36 (D) 12 (E) 18

3. Triangle ABC is constructed with $\angle ACB = 120°$ and $\angle CAB = 40°$. AC is extended to P so that $AP = AC + 2CB$. Determine the measure of $\angle ABP$.

4. Two perpendicular diameters are drawn in a circle of radius 2. All possible chords parallel to and at a distance of 1 unit from these diameters are drawn. What is the sum of the lengths of the six chords?

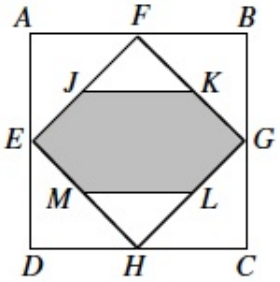
5. An *ascending* integer occurs when each digit is greater than any digit that preceeds it. An example is 478. How many ascending integers occur between 200 and 300?

6. Of 45 students in a mathematics class, 27 own a bicycle and 22 own a skateboard. Three students do not own either one. How many students own both a bicycle and a skateboard?

7. In the diagram, the four circles have a common centre, and have radii of 1, 2, 3 and 4. The ratio of the area of the shaded regions to the area of the largest circle is

(A) 5:8 (B) 1:4 (C) 7:16 (D) 1:2 (E) 3:8

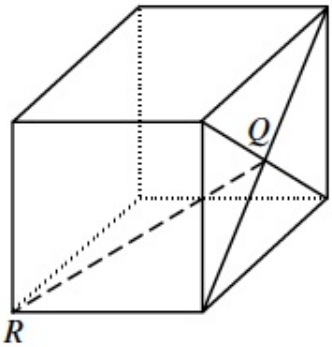
8. The area of square $ABCD$ is 64. The midpoints of its sides are joined to form the square $EFGH$. The midpoints of its sides are J, K, L and M. The area of the shaded region is



(A) 32 (B) 24 (C) 20 (D) 28 (E) 16

9. Using only digits 1, 2, 3, 4 and 5, a sequence is created as follows: one 1, two 2's, three 3's, four 4's, five 5's, six 1's, seven 2's, and so on.
the sequence appears as: 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 1, 1, 1, 1, 1, 1, 2, 2, ...
The 100th digit in the sequence is
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

10. Q is the point of intersection of the diagonals of one face of a cube whose edges have length 2 units. The length of QR is



(A) 2 (B) $\sqrt{8}$ (C) $\sqrt{5}$ (D) $\sqrt{12}$ (E) $\sqrt{6}$

Answer Key

Problem Set A

1. (D) 13
2. (A) 8
3. (D) 13
4. (B) 84
5. (D) 66°
6. (D) 3
7. (E) $(2, 4)$
8. (D) 70
9. (E) 70
10. (B) 20
11. (E) 43%
12. (D) 30

Narrowing Down Choices

1. (A) 15
2. (B) 24

Plug in the given values

1. (B) $\frac{x}{2}$
2. (A) $\frac{ab-1}{2}$

Try writing out a few cases

3. (B) a multiple of 6

Problem Set B

1. (B) 50 and 100
2. (E) 10
3. (B) -1
4. (E) 190
5. (D) 42
6. (E) \$9.67
7. (D) 30
8. (E) 14
9. (D) 80
10. (D) 4
11. (A) 5
12. (C) 40°
13. (C) 2
14. (E) $\frac{40}{3}$
15. (A) 18

Problem Set C

1. (C) $2(x^2 + 9)$
2. (B) 9
3. 110°
4. $8 + 8\sqrt{3}$
5. 21
6. 7
7. (E) $3 : 8$
8. (B) 24
9. (D) 4
10. (E) $\sqrt{6}$