



University of Waterloo  
Faculty of Mathematics



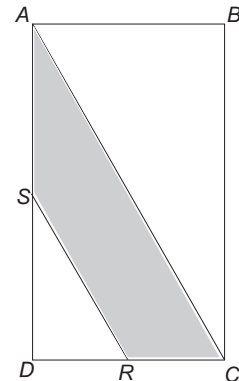
Centre for Education in  
Mathematics and Computing

## Junior Math Circles March 31, 2010 Gauss Preparation

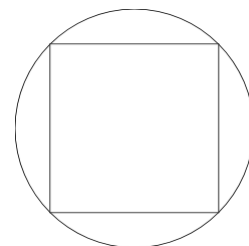
### Problem Set 1

#### Geometry

- The length of a large rectangular room is 7 metres more than its width. If the perimeter is 34 metres, the length of the room, in metres, is:  
A) 4            B) 13.5            C) 19            D) 12            E) 6.75
- One of the angles in an isosceles triangle is equal to  $40^\circ$ . What is the measure of the largest angle that this triangle could contain?  
A) 40            B) 70            C) 90            D) 100            E) 110
- A rectangular prism with integer side lengths has two faces with areas 15 and 21. What could its volume be?  
A) 27            B) 45            C) 63            D) 105            E) 142
- In rectangle  $ABCD$ ,  $S$  and  $R$  are midpoints of sides  $AD$  and  $CD$  respectively. If  $AB = 4$  and  $BC = 8$ , what is the area of the shaded region  $ASRC$ ?  
A) 8            B) 12            C) 14  
D) 16            E) 20



- A square inscribed in a circle is shown to the right. If the area of the square is  $49 \text{ cm}^2$ , the radius of the circle, in cm, is  
A) 7            B) 14            C)  $\frac{7\sqrt{2}}{2}$   
D)  $\frac{49}{8}$             E)  $\frac{\sqrt{28}}{2}$



Number Theory

- Which of the following numbers is not divisible by 6?  
A) 570      B) 198      C) 354      D) 684      E) 482
- The product of two whole numbers is 24. The smallest possible sum of these two numbers is  
A) 9      B) 10      C) 11      D) 14      E) 25
- The smallest value of  $n$  which will make  $12n$  divisible by 28 is  
A) 14      B) 4      C) 84      D) 7      E) none of these
- A two-digit number is divisible by 8, 12 and 18. The number is between  
A) 10 and 19    B) 20 and 39    C) 40 and 59    D) 60 and 79    E) 80 and 99
- Each of the integers 226 and 318 have digits whose product is 24. How many three-digit positive integers have digits whose product is 24?  
A) 4      B) 18      C) 24      D) 12      E) 21

Probability/Data Management

- Five students wrote a test and four of the marks were 70, 78, 84, and 90. If the median was 78, what is the highest possible average(mean)?  
A) 84.4      B) 78      C) 81      D) 79.8      E) 80
- When two ordinary dice, one black and one white, are rolled, the number of ways the sum of 7 can occur is  
A) 2      B) 8      C) 4      D) 7      E) 6
- Five students wrote a mathematics test. The average mark was 68. If the marks for four of the students were 75, 62, 84 and 53, the mark of the fifth student was  
A) 66      B) 68      C) 76      D) 68.5      E) 56
- A bottle of childrens vitamins contains tablets in three different shapes. Among the vitamins, there are 60 squares, 60 triangles and 60 stars. Each shape comes in an equal number of three different flavours strawberry, grape, and orange. A tablet is randomly chosen from a newly opened bottle. What is the probability that the tablet is a grape star?  
A)  $\frac{1}{9}$       B)  $\frac{1}{60}$       C)  $\frac{1}{20}$       D)  $\frac{1}{3}$       E)  $\frac{1}{180}$

5. Mark has a bag that contains 3 black marbles, 6 gold marbles, 2 purple marbles, and 6 red marbles. Mark adds a number of white marbles to the bag and tells Susan that if she now draws a marble at random from the bag, the probability of it being black or gold is  $\frac{3}{7}$ . The number of white marbles that Mark adds to the bag is
- A) 5            B) 2            C) 6            D) 4            E) 3

### Algebra

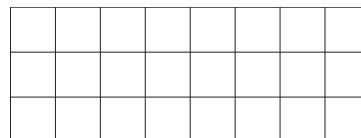
1. What percentage of 120 is 78?  
A) 60%            B) 65%            C) 66%            D) 70%            E) 75%
2. If  $a = 2$ ,  $b = 3$ , and  $c = 4$  then  $bca$  equals  
A) 32            B) 24            C) 144            D) 36            E) 48
3. An expression that can be placed in the box to make the equation  $\frac{3 \times 6 \times 9}{3} = \frac{?}{2}$  true is  
A)  $2 \times 4 \times 6$     B)  $3 \times 4 \times 6$     C)  $2 \times 6 \times 9$     D)  $2 \times 4 \times 8$     E)  $2 \times 12 \times 18$
4. A dog and a rabbit are 160 metres apart. The dog chases the rabbit. For every 9 metres the dog runs, the rabbit jumps 7 metres. The distance, in metres, the dog must run to overtake the rabbit is  
A) 240            B) 720            C) 880            D) 560            E) 320
5. A 250 m train travels through a 2km tunnel at 27 km/hr. How long is it from the time the front of the train enters the tunnel, to when it completely leaves it?  
A) 5 minutes    B)  $\frac{2}{27}$  hours    C) 30 minutes    D) 5 seconds    E) 12 minutes

### Miscellaneous

1. One integer that is greater than -6 is  
A) -7            B)  $\frac{1}{2}$             C)  $4 - 10$             D)  $-\frac{1}{2}$             E) -5
2. A sequence is 1, 2, 5, 10, 17, ... . A possible seventh number in this sequence is  
A) 24            B) 26            C) 37            D) 50            E) 33

3. How many squares of all sizes are there in the diagram?

- A) 44            B) 30            C) 24  
D) 6            E) 20



4. A palindrome is a number which remains the same when its digits are written in reverse order. For example, 131 is a palindrome. A cars odometer reads 15951. The least number of kilometres required for the next palindrome to appear is  
 A) 210            B) 1010            C) 110            D) 1110            E) 10111
5. A different letter is painted on each face of a cube. This cube is shown below in 3 different positions:



What letter belongs on the shaded face of this cube in the following diagram?

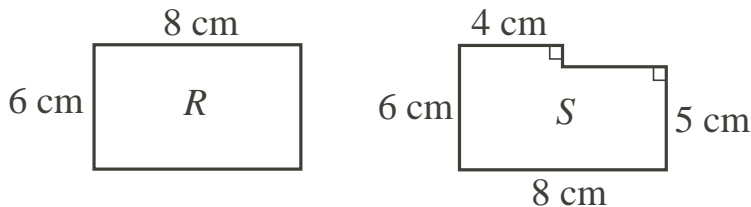
- A) T            B) P            C) X  
 D) E            E) V



## Problem Set 2

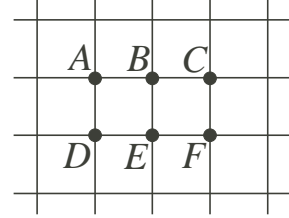
### Geometry

1. Kalyn cut rectangle  $R$  from a sheet of paper. A smaller rectangle is then cut from the large rectangle  $R$  to produce figure  $S$ . In comparing figure  $S$  to figure  $R$ :

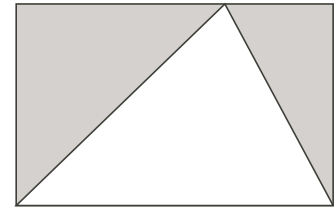


- A) the area and perimeter both decrease  
 B) the area decreases and the perimeter increases  
 C) the area and perimeter both increase  
 D) the area increases and the perimeter decreases  
 E) the area decreases and the perimeter stays the same
2. A cube has a volume of  $125 \text{ cm}^3$ . What is the area of one face of the cube?  
 A)  $20 \text{ cm}^2$             B)  $25 \text{ cm}^2$             C)  $41\frac{2}{3} \text{ cm}^2$             D)  $5 \text{ cm}^2$             E)  $75 \text{ cm}^2$

3. Six points  $A, B, C, D, E$  and  $F$  are placed on a square grid, as shown. How many triangles that are *not* right-angled can be drawn using 3 of these 6 points as vertices?

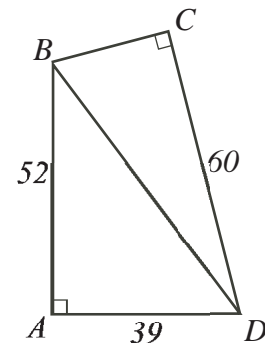


- A) 2                      B) 1                      C) 6  
D) 0                      E) 4
4. In the diagram, the rectangle has length 11 and width 7. The area of the shaded part is



- A)  $\frac{77}{2}$                       B) 37.5                      C) 18  
D) 36                      E) none of these
5. One angle of a triangle is twice the size of the second angle, and the third angle is  $66^\circ$ . The smallest angle, in degrees, is

- A) 66                      B) 33                      C) 38                      D) 24                      E) 22
6. The quadrilateral  $ABCD$  is made of two right triangles  $ABD$  and  $BCD$ , which share their hypotenuse  $BD$ . If  $AB = 52$ ,  $AD = 39$  and  $CD = 60$ , then what is the length of  $BC$ ?



- A) 25                      B) 30                      C) 45  
D) 65                      E) 88

### Algebra

1. Juan and Mary play a two-person game in which the winner gains 2 points and the loser loses 1 point. If Juan won exactly 3 games and Mary had a final score of 5 points, how many games did they play?
- A) 7                      B) 8                      C) 4                      D) 5                      E) 11
2. When a number is divided by 7, it gives a quotient of 4 with a remainder of 6. What is the number?
- A) 17                      B) 168                      C) 34                      D) 31                      E) 46
3. Karl had his salary reduced by 10%. He was later promoted and his salary was increased by 10%. If his original salary was \$20 000, what is his present salary?
- A) \$16 200                      B) \$19 800                      C) \$20 000                      D) \$20 500                      E) \$24 000

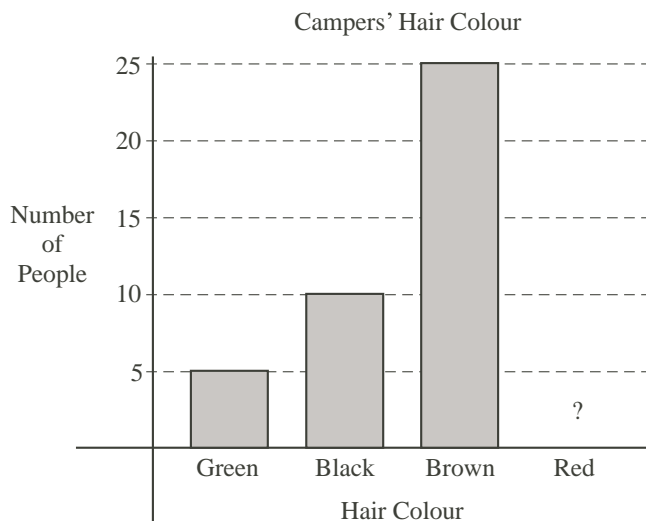
4. Which of the following fractions has the largest value?  
 A)  $\frac{8}{9}$       B)  $\frac{7}{8}$       C)  $\frac{66}{77}$       D)  $\frac{55}{66}$       E)  $\frac{4}{5}$
5. Nine coins have a value of \$1.35. If the coins are quarters and dimes, the number of dimes is  
 A) 2      B) 5      C) 7      D) 1      E) 6
6. What is  $1 - 2 + 3 - 4 + 5 - 6 + \dots + 17 - 18 + 19 - 20$  equal to?  
 A)  $-20$       B)  $-15$       C)  $-12$       D)  $-10$       E)  $-8$

### Number Theory

1. A prime number is called a “Superprime” if doubling it, and then subtracting 1, results in another prime number. The number of Superprimes less than 15 is  
 A) 2      B) 3      C) 4      D) 5      E) 6
2. If  $P = 1000$  and  $Q = 0.01$ , which of the following calculations gives the largest result?  
 A)  $P + Q$       B)  $P \times Q$       C)  $\frac{P}{Q}$       D)  $\frac{Q}{P}$       E)  $P - Q$
3. The smallest value of  $k$  so that  $60k$  is a perfect square is  
 A) 60      B) 15      C) 6      D) 45      E) 9
4. The number 6 has exactly four positive divisors: 1, 2, 3 and 6. What is the smallest positive integer that has exactly *five* divisors?  
 A) 8      B) 9      C) 12      D) 16      E) 18

### Data

1. The bar graph shows the hair colours of the campers at Camp Gauss. The bar corresponding to redheads has been accidentally removed. If 50% of the campers have brown hair, how many of the campers have red hair?  
 A) 5      B) 10      C) 25  
 D) 50      E) 60



2. Naoki wrote nine tests, each out of 100. His average on these nine tests is 68%. If his lowest mark is omitted, what is his highest possible resulting average?  
 A) 76.5%      B) 70%      C) 60.4%      D) 77%      E) 76%
3. Five people are in a room for a meeting. When the meeting ends, each person shakes hands with each of the other people in the room exactly once. The total number of handshakes that occurs is  
 A) 5      B) 10      C) 12      D) 15      E) 25
4. Abraham's mathematics exam had 30 algebra questions and 50 geometry questions, each worth 1 mark. He got 70% of the algebra questions correct, and his overall exam mark was 80%. How many geometry questions did he answer correctly?  
 A) 43      B) 45      C) 39      D) 41      E) 35

### Miscellaneous

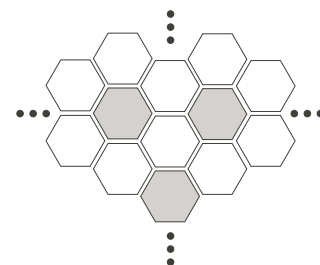
1. In a "Fibonacci" sequence of numbers, each term beginning with the third, is the sum of the previous two terms. The first number in such a sequence is 2 and the third is 9. What is the eighth term in the sequence?  
 A) 34      B) 36      C) 107      D) 152      E) 245
2. Paul, Quincy, Rochelle, Surinder and Tony are sitting around a table. Quincy sits in the chair between Paul and Surinder. Tony is not beside Surinder. Who is sitting on either side of Tony?  
 A) Paul and Rochelle      B) Quincy and Rochelle      C) Paul and Quincy  
 D) Surinder and Quincy      E) Not possible to tell
3. A fog horn sounds a blast for 2 seconds and is silent for 8 seconds. In  $3\frac{1}{2}$  hours, the number of blasts it makes is  
 A) 21      B) 126      C) 1260      D) 6300      E) 3150
4. At a party, exactly 15 people ate hot dogs, and exactly 12 people ate hamburgers. Ten of them ate both. Three people ate neither. The number of people at the party was  
 A) 20      B) 40      C) 35      D) 30      E) 18
5. If Bob drove 180 km from Toronto to London at 90 km/h, then drove back at only 60 km/h, what was his average speed over the entire trip?  
 A) 70 km/h      B) 72 km/h      C) 75 km/h      D) 78 km/h      E) 84 km/h

## Part C Challenge

The following problems are some of the hardest problems that have appeared on the Grade 7 and Grade 8 Gauss contests in the past. All of the problems are taken from Part C, the last 5 questions of the contests, and so they are good practice for that section.

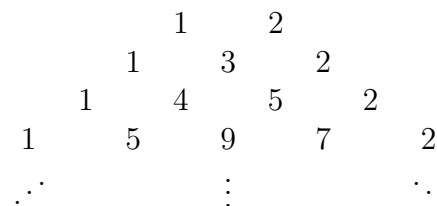
1. (*G7, 2000*) A square floor is tiled, as partially shown, with a large number of regular hexagonal tiles. The tiles are coloured blue or white. Each blue tile is surrounded by 6 white tiles and each white tile is surrounded by 3 white and 3 blue tiles. Ignoring part tiles, the ratio of the number of blue tiles to the number of white tiles is closest to

- A) 1 : 6                      B) 2 : 3                      C) 3 : 10  
D) 1 : 4                      E) 1 : 2



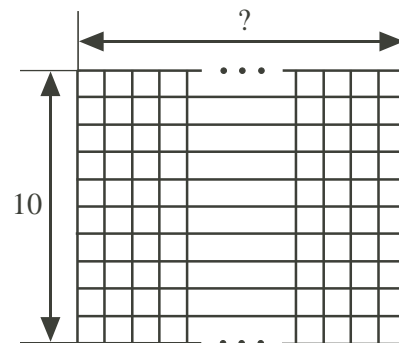
2. (*G7, 2003*) In the pattern of numbers shown, every row begins with a 1 and ends with a 2. Each of the numbers, not on the end of a row, is the sum of the two numbers located immediately above and to the right, and immediately above and to the left. For example, in the fourth row the 9 is the sum of the 4 and the 5 in the third row. If this pattern continues, the sum of all the numbers in the thirteenth row is

- A) 12 270                      B) 12 276                      C) 12 282  
D) 12 288                      E) 12 294



3. (*G7, 2004*) A grid with 10 rows and some number of columns is made up of unit squares, as shown. A domino ( $\square\square$ ) can be placed horizontally or vertically to exactly cover two unit squares. There are 2004 positions in which the domino could be placed. The number of columns in the grid is

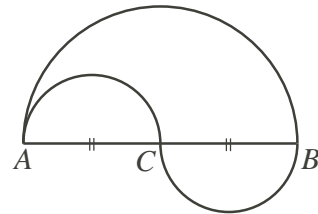
- A) 105                      B) 106                      C) 107  
D) 108                      E) 109



4. (*G7, 2006*) Five students wrote a quiz with a maximum score of 50. The scores of four of the students were 42, 43, 46, and 49. The score of the fifth student was  $N$ . The average (mean) of the five students' scores was the same as the median of the five students' scores. The number of values of  $N$  which are possible is
- A) 3                      B) 4                      C) 1                      D) 0                      E) 2



5. (*G8, 1999*) In the diagram,  $AC = CB = 10$  m, where  $AC$  and  $CB$  are each the diameter of the small equal semi-circles. The diameter of the larger semi-circle is  $AB$ . In travelling from  $A$  to  $B$ , it is possible to take two paths. One path goes along the semi-circular arc from  $A$  to  $B$ . A second path goes along the semi-circular arc from  $A$  to  $C$  and then along the semi-circular arc from  $C$  to  $B$ . The difference in the lengths of these two paths is



- A)  $12\pi$       B)  $6\pi$       C)  $3\pi$   
 D)  $2\pi$       E) 0

6. (*G8, 2000*) The cookies in a jar contain a total of 1000 chocolate chips. All but one of these cookies contains the same number of chips; it contains one more chip than the others. The number of cookies in the jar is between one dozen and three dozen. What is the sum of the number of cookies in the jar and the number of chips in the cookie with the extra chocolate chip?

- A) 65      B) 64      C) 63      D) 66      E) 67

7. (*G8, 2001*) Tony and Maria are training for a race by running all the way up and down a 700 m long ski slope. They each run up the slope at different constant speeds. Coming down the slope, each runs at double his/her uphill speed. Maria reaches the top first and immediately starts running back down, meeting Tony 70 m from the top. When Maria reaches the bottom, how far behind is Tony?

- A) 140 m      B) 250 m      C) 280 m      D) 300 m      E) 320 m

8. (*G8, 2002*) If each diagonal of a square has length 2, the area of the square is

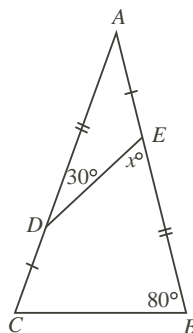
- A) 1      B) 2      C) 3      D) 4      E) 5

9. (*G8, 2004*) Water is poured from a full 1.5 L bottle into an empty glass until both the glass and the bottle are  $\frac{3}{4}$  full. What is the volume of the glass?

- A) 0.5 L      B) 0.75 L      C) 1.125 L      D) 0.6 L      E) 0.4 L

10. (*G8, 2004*) In the diagram, the value of  $x$  is

- A) 40      B) 45      C) 50  
 D) 55      E) 60



## Answers

### Problem Set 1

#### Geometry

1. D
2. D
3. D
4. B
5. C

#### Number Theory

1. E
2. B
3. D
4. D
5. E

#### Probability & Data

1. E
2. E
3. A
4. A
5. D

#### Algebra

1. B
2. B
3. C
4. B

5. A

#### Miscellaneous

1. E
2. C
3. A
4. C
5. E

### Problem Set 2

#### Geometry

1. E
2. B
3. E
4. A
5. C

6. A

#### Algebra

1. A
2. C
3. B
4. A
5. E
6. D

#### Number Theory

1. B
2. C

3. B

4. D

#### Data

1. B
2. A
3. B
4. A

#### Miscellaneous

1. C
2. A
3. C
4. A
5. B

### Challenge

1. E
2. D
3. B
4. A
5. E
6. A
7. D
8. B
9. A
10. C