



## Intermediate Math Circles for Wednesday 20 October 2010

### 1. Opening Problem

Teams A, B, C, and D competed against each other once. The results table was as follows:

Team	Wins	Ties	Losses	Goals for	Goals against
A	3	0	0	5	1
B	1	1	1	2	2
C	0	2	1	5	6
D	0	1	2	3	6

You went to the final match, when B played C and the score was 1-1.

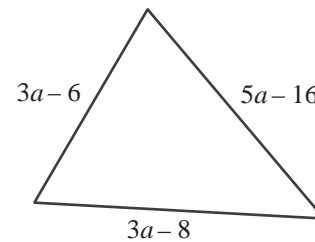
- Which team won in each of the six matches?
- What were the scores in the matches that B played?
- What were the scores in the rest of the matches?



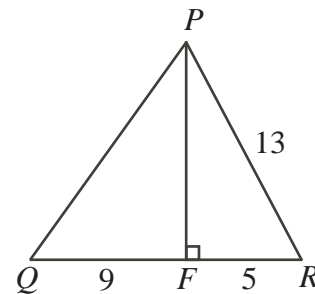
## Intermediate Math Circles for Wednesday 20 October 2010

### 2. Intermediate Week 2 Problem Set 1: Solving Some More Problems

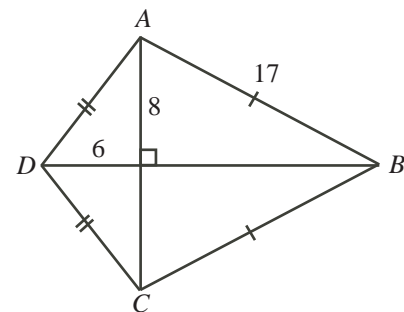
- Determine the number of different values of  $a$  for which the given triangle is isosceles.



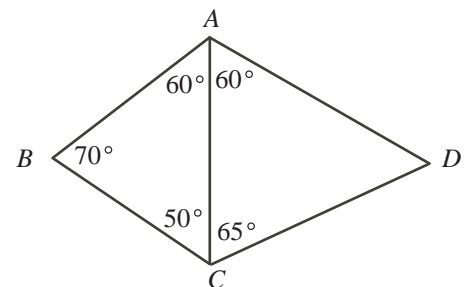
- In triangle  $PQR$ ,  $F$  is the point on  $QR$  so that  $PF$  is perpendicular to  $QR$ . If  $PR = 13$ ,  $RF = 5$ , and  $FQ = 9$ , what is the perimeter of  $\triangle PQR$ ?



- Calculate the area of figure  $ABCD$ .



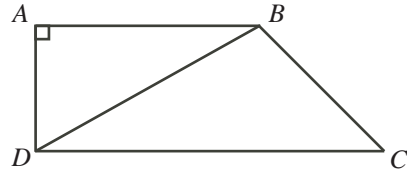
- In the diagram, which side is the longest:  $AB$ ,  $BC$ ,  $AC$ ,  $CD$ , or  $AD$ ?



- If a 3 m stake casts a shadow 7 m long, what is the height of a tree that casts a shadow 63 m long?

6. A *scalene* triangle is a triangle whose side lengths are all different. Determine the side lengths of all possible scalene triangles with integer side lengths and perimeter less than 13.

7. In the diagram,  $AB = 4$ ,  $DC = 6$ , and  $AB$  is parallel to  $DC$ . If  $\angle C = 45^\circ$ , determine the length of  $BD$ .

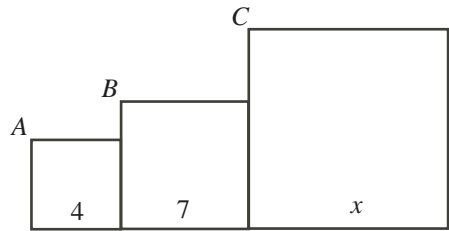


8. Triangle  $ABC$  is similar to  $\triangle XYZ$ . If  $AB = 4$ ,  $YZ = 9$ , and  $BC = XY = p$ , determine the value of  $p$ .

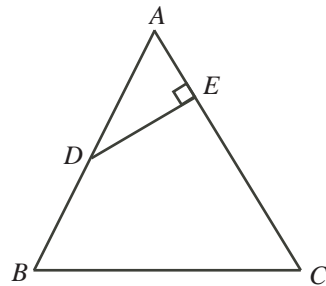
9. A triangle can be formed having side lengths 4, 5 and 8. It is impossible however, to construct a triangle with side lengths 4, 5 and 10. Using the side lengths 2, 3, 5, 7 and 11, how many different triangles *with exactly two equal sides* can be formed?

10. A triangle can be formed having side lengths 4, 5 and 8. It is impossible however, to construct a triangle with side lengths 4, 5 and 10. Ron has eight sticks, each having an integer length. He observes that he cannot form a triangle using any three of these sticks as side lengths. What is the shortest possible length of the longest of the eight sticks?

11. In the adjacent squares shown, the vertices  $A$ ,  $B$  and  $C$  lie in a straight line. What is the value of  $x$ ?

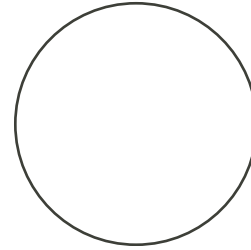


12. In the diagram,  $AD = BD = 5$ ,  $EC = 8$  and  $AE = 4$ . Determine the length of  $BC$ .



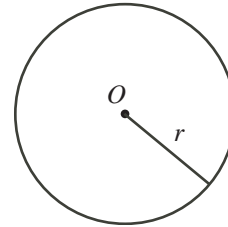
### 3. Six Facts About Circles

CF1. Circles are round.



CF2. A circle is determined by its centre and its radius.

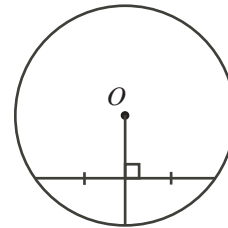
A circle is the set of points at a fixed distance from a fixed point.



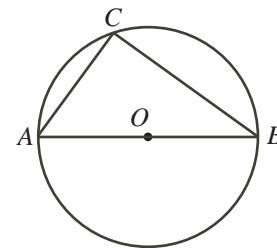
CF3. A chord is a line segment that begins and ends on a circle.

If a radius cuts a chord into two equal parts, then the radius is perpendicular to the chord.

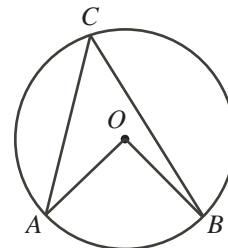
If a radius is perpendicular to a chord, then it cuts the chord into two equal parts.



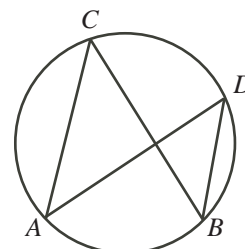
CF4. If  $AB$  is a diameter, then  $\angle ACB = 90^\circ$ .



CF5. If  $AB$  is a chord and  $C$  is on the major arc, then  $\angle AOB = 2\angle ACB$ .



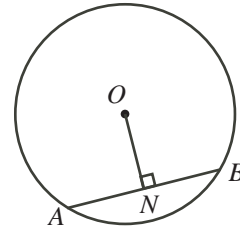
CF6. If  $AB$  is a chord and  $C$  and  $D$  are on the major arc, then  $\angle ACB = \angle ADB$ .



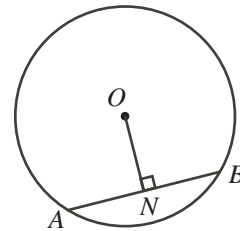
## Intermediate Math Circles for Wednesday 20 October 2010

### 4. Intermediate Week 3 Problem Set 1: Solving Some More Problems

1. Determine the length of the chord  $AB$  if  $OA = 5$  and  $ON = 3$ .

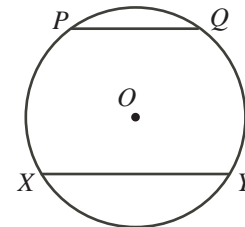


2. If  $AB = 10$  and  $OA = 13$ , determine the length of  $ON$ .

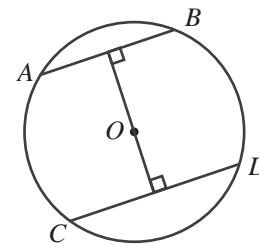


3. A circle has a diameter of length 26. If a chord of the same circle has a length of 10, how far is the chord from the centre?

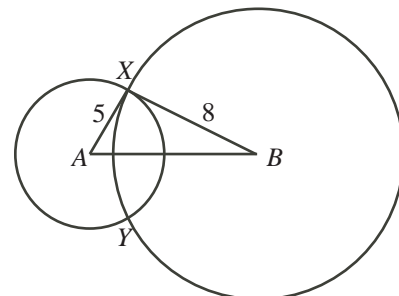
4. Calculate the distance between the parallel chords  $PQ$  and  $XY$  if  $PQ = 6$ ,  $XY = 8$ , and the radius of the circle is 5.



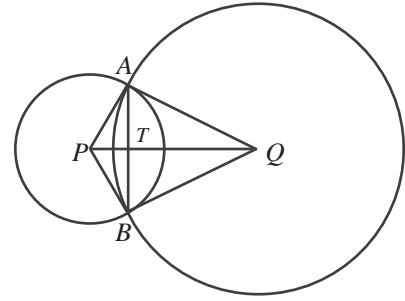
5. The two parallel chords  $AB$  and  $CD$  are a distance of 14 part. If  $AB$  has length 12 and the radius of the circle is 10, calculate the length of  $CD$ .



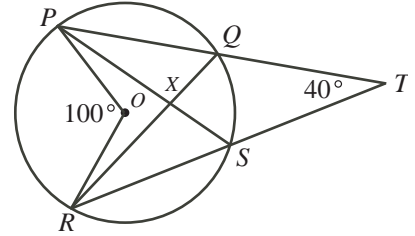
6. Two circles with centre  $A$  and  $B$  have radii 5 and 8, respectively. The circles intersect at the points  $X$  and  $Y$ . If  $XY = 8$ , determine the length of  $AB$ , the distance between the centres.



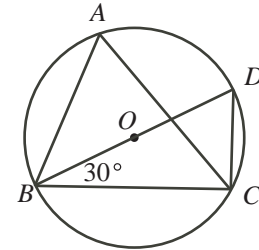
7. In the diagram,  $PA = 13$  and  $QA = 20$ , where  $P$  and  $Q$  are the centres of the circles. Determine the length of  $AB$  if  $PQ = 21$ .



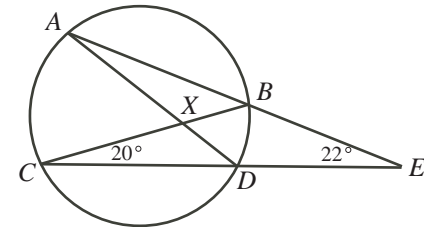
8. In the diagram,  $O$  is the centre of the circle. Determine the measure of  $\angle QXS$ .



9. Determine the measure of  $\angle BAC$ .

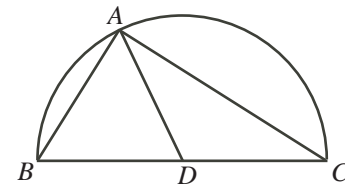


10. Determine the measure of  $\angle ADC$  and of  $\angle AXB$ .

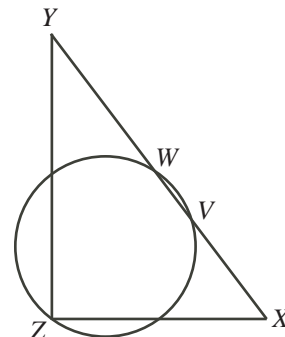


11. A cyclic quadrilateral is a quadrilateral that has all four its vertices on the same circle. Prove that opposite angles are supplementary.

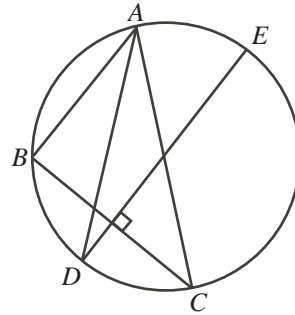
12. In the diagram,  $\triangle ABC$  is inscribed in the semicircle with centre  $D$ . If  $AB = AD$ , determine the measure of  $\angle ACD$ .



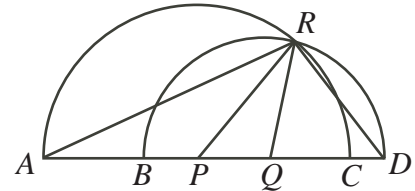
13. In the diagram,  $\triangle XYZ$  is right-angled at  $Z$ .  $W$  is the midpoint of  $XY$ , and the circle with diameter  $ZW$  intersects  $WX$  at  $V$ . If  $XY = 50$  and  $WV = 7$ , determine the length of  $XZ$ .



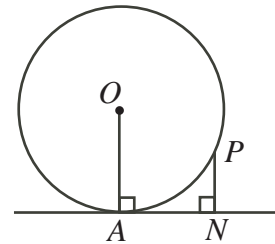
14.  $\triangle ABC$  has its vertices on a circle, as shown. The bisector of the angle at  $A$  meets the circumference at  $D$ . From  $D$ , a line is drawn perpendicular to the chord  $BC$  so that it meets the circumference at  $E$ . Prove that  $DE$  is a diameter of the circle.



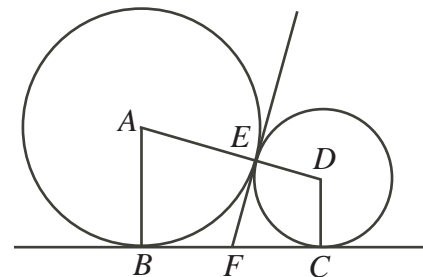
15. In the diagram, points  $B, P, Q,$  and  $C$  lie on line segment  $AD$ . The semi-circle with diameter  $AC$  has centre  $P$  and the semi-circle with diameter  $BD$  has centre  $Q$ . The two semi-circles intersect at  $R$ . If  $\angle PRQ = 40^\circ$ , determine the measure of  $\angle ARD$ .



16. In the diagram,  $O$  is the centre of the circle,  $AN$  is tangent to the circle at  $A$ ,  $P$  lies on the circle, and  $PN$  is perpendicular to  $AN$ . If  $AN = 15$  and  $PN = 9$ , determine the radius of the circle.



17. In the diagram,  $O$  is the centre of the circle,  $AN$  is tangent to the circle at  $A$ ,  $P$  lies on the circle, and  $PN$  is perpendicular to  $AN$ . If  $AN = 15$  and  $PN = 9$ , determine the radius of the circle.



18. If  $O$  is the centre of the circle and  $\angle BCD = 82^\circ$ , what is the value of  $x$  in degrees?

