Sequences and Series

Sequences and Series - week 3

This week we didn't introduce any new material on Sequences and Series. We looked at some solutions to some of Week Two's questions, and then went on to look at some new problems for week 3.

What follows are some facts which will assist you in completing the questions. Final answers, not complete solutions, will be posted shortly.

Week Two Questions

6. To be able to complete the question you need to recall that, in a perfect square all prime factors must occur an even number of times.

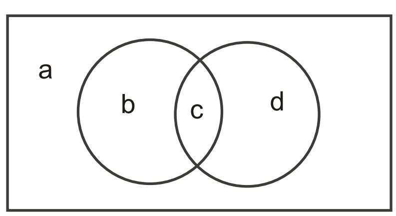
8. In part (a), the given information will result in a quadratic equation, which is factorable, to find the values of the common ration, r. From there you can determine the initial terms and thus the sequence.
   Note that in part (b) you do not want the sum of all the terms in the series, but only of those terms which are less than 1.

12. Look at groups of five terms of the sequence.

Week Three Questions

2. Determine the sum of all the integers from 1 to 4000, then the sum of the multiples of 5 from 1 to 4000, and subtract the second sum from the first.

3. This is similar to question 2, but slightly more complicated. Consider the Venn Diagram below.



If we imagine putting each of the positive integers from 1 to $10n$ in the rectangle as follows:
all multiples of 2 go somewhere in the left circle, so regions b and c,
all multiples of 5 go somewhere in the right circle, so regions c and d,

all other integers go outside of both circles, region a.

If we do this properly, the integers in region c will be multiples of both 2 and 5, and are thus multiples of 10.
We are asked to determine how many integers are in region a,outside both circles.
To do this we calculate the sum of all the numbers from 1 to $10n$, call this sum S.
Subtract from S the sum of all multiples of 2 and the sum of all multiples of 5 from 1 to $10n$.
That is, sum all the numbers in the rectangle, subtract the sum of all the numbers in regions b and c, and the sum of all the numbers in regions c and d. We have now subtracted the sum of the numbers in region c twice, and so we must add that sum back in to get the required sum. The integers in region c are simply the multiples of 10 from 1 to $10n$, so obtain this sum and add it back.

5. We need a *partial fraction decomposition* to help us solve this problem. Without going into details as to how we get this, show that

$$\frac{1}{(n+1)(n+2)} = \frac{1}{n+1} - \frac{1}{n+2}.$$

This can be used to break each term in the sum into two parts, and the result is a *telescoping series* which we can use to find the sum of any finite number of terms, beginning with the first term, of the series.

6 and 7 Techniques from week 1 can be used to solve these questions.