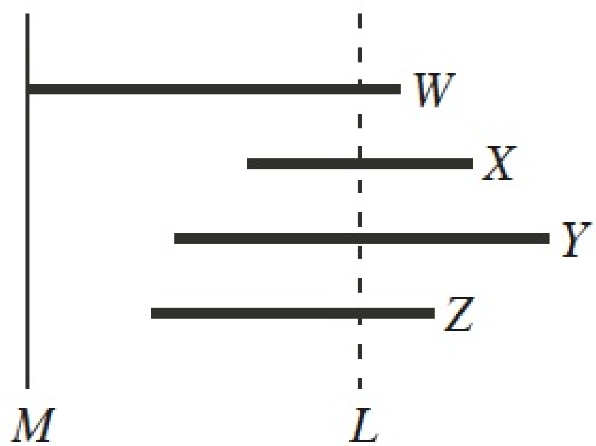
University of Waterloo
Faculty of Mathematics

Centre for Education in
Mathematics and Computing

Intermediate Math Circles
November 17, 2010
Equations and Inequalities with One Variable

Problem:
Four pieces of lumber are placed in parallel positions, as shown, perpendicular to line M:

- Piece W is 5 m long
- Piece X is 3 m long and its left end is 3 m from line M
- Piece Y is 5 m long and is 2 m from line M
- Piece Z is 4 m long and is 1.5 m from line M



A single cut, perpendicular to the pieces of lumber, is made along the dotted line L. The total length of lumber on each side of L is the same. What is the length, in metres, of the part of piece W to the left of the cut?

Inequalities:

$a < b$ means:
a is strictly less than b
a is to the left of b on the number line
$b = a + p$ where p is some positive real number

Solving Inequalities in One Variable:

Rules for Inequalities:

1: Adding any number to both sides of an inequality preserves the inequality.
If $a < b$, then $a + c < b + c$.

2: Multiplying or dividing both sides of an inequality by a positive number preserves the inequality.
If $a < b$ and $c > 0$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.

3: Multiplying both sides of an inequality by a negative number changes the direction of the inequality.
If $a < b$ and $c < 0$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.

4: If $0 < a < b$, then $a^2 < b^2$.

5: If $0 < a < b$, then $\frac{1}{a} > \frac{1}{b}$.

Problem Set

1. The average of a set of n integers is 10. If we remove the integer 2 from this set, the average of the remaining integers is 14. What is the value of n?

2. In a bin at the Cayley Convenience Store, there are 200 candies. Of these candies, 90% are black and the rest are gold. After Matilda eats some of the black candies, 80% of the remaining candies in the bin are black. How many black candies did Matilda eat?

3. The five expressions $2x+1$, $2x-3$, $x+2$, $x+5$ and $x-3$ can be arranged in a different order so that the sum of the first three expressions is $4x+3$ and the sum of the last three expressions is $4x+4$. What is the middle expression in the new list?

4. Solve $5x-2 \leq 3x-10$ and sketch your solution.

5. Solve $10-7x < -4x-9$ and sketch your solution.

6. Solve $-\frac{1}{2}(2+5x) \geq \frac{2}{3}(15-3x)$ and sketch your solution.

7. How many integer values of x satisfy $\frac{x-1}{3} < \frac{5}{7} < \frac{x+4}{5}$?

8. How many positive integers p satisfy $-1 < \sqrt{p} - \sqrt{100} < 1$?

9. If $-2 < x < 3$ then determine a and b in $a < 2x+3 < b$.

10. What values of x satisfy the inequality $-3 < 5 - \frac{2}{x} < 3$? Sketch your solution.

11. Solve $2 - \frac{1}{x} < 3$ and sketch your solution.

12. Solve $\frac{2}{x} + 3 \geq 4$ and sketch your solution.

13. The front wheel of Georgina's bicycle has a diameter of 0.75 metres. She cycled for 6 minutes at a speed of 24 kilometres per hour. How many complete rotations did the wheel made during this time?

14. A computer software retailer has 1200 copies of a new software package to sell. From past experience, she knows that:

    - Half of them will sell right away at the original price she sets,
    - Two-thirds of the remainder will sell later when the price is reduced by 40%, and
    - The remaining copies will sell in a clearance sale at 75% off the original price.

    In order to make a reasonable profit, the total sales revenue must be greater than or equal to \$72 000. To the nearest cent, what is the smallest original price she should set?