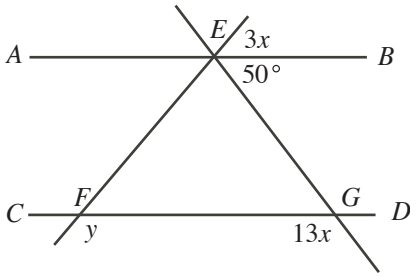
WATERLOO
MATHEMATICS
University of Waterloo
Faculty of Mathematics

Centre for Education in
Mathematics and Computing

Intermediate Math Circles for Wednesday 13 October 2010

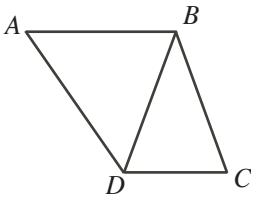
1. Intermediate Week 1 Problem Set 1: Solving More Problems

1. In the diagram, AB is parallel to CD. Determine the values of x and y.

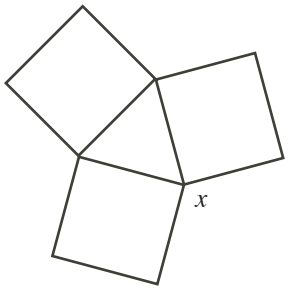


2. Triangle ABC has a right angle at B. AC is extended to D so that $CD = CB$. The bisector of angle A meets BD at E. Prove that $\angle AEB = 45°$.

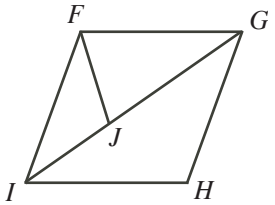
3. In the diagram, AB is parallel to DC and $AB = BD = BC$. If $\angle A = 52°$, determine the measure of $\angle DBC$.



4. The diagram shows three squares of the same size. What is the value of x?

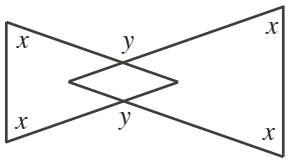


5. The diagram shows a rhombus $FGHI$ and an isosceles triangle FGJ in which $GF = GJ$. Angle FJI equals $111°$. What is the measure of angle JFI?

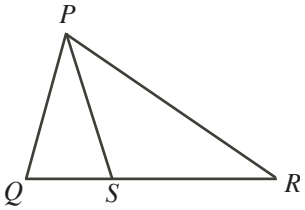


6. $ABCD$ is a square. The point E is outside the square so that CDE is an equilateral triangle. Find angle BED.

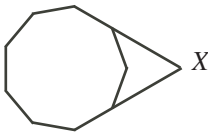
7. The diagram shows two isosceles triangles in which the four angles marked x are equal. The two angles marked y are also equal. Find an equation relating x and y.



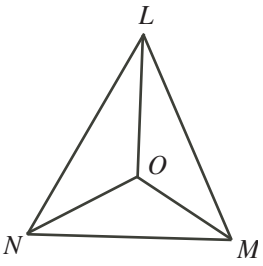
8. In the diagram, QSR is a straight line. $\angle QPS = 12°$ and $PQ = PS = RS$. What is the size of $\angle QPR$?



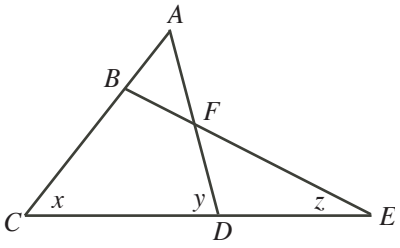
9. The diagram shows a regular nonagon with two sides extended to meet at point X. What is the size of the acute angle at X?



10. The three angle bisectors of triangle LMN meet at a point O as shown. Angle LNM is 68°. What is the size of angle LOM?



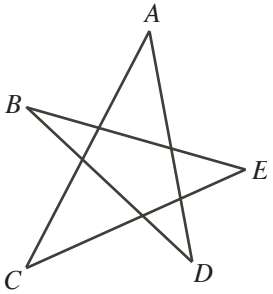
11. In the figure shown, $AB = AF$ and ABC, AFD, BFE, and CDE are all straight lines. Determine an equation relating x, y and z.



12. The angles of a nonagon are nine consecutive numbers. What are these numbers?

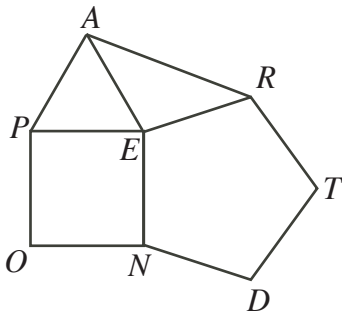
13. What is the measure of the angle formed by the hands of a clock at 9:10?

14. Determine the sum of the angles A, B, C, D, and E in the five-pointed star shown.

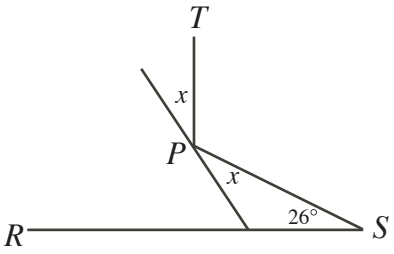


15. In $\triangle PQR$, $PQ = PR$. PQ is extended to S so that $QS = QR$. Prove that $\angle PRS = 3(\angle QSR)$.

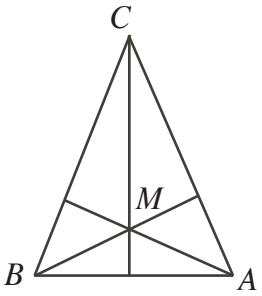
16. A regular pentagon is a five-sided figure which has all of its angles equal and all of its side lengths equal. In the diagram, $TREND$ is a regular pentagon, PEA is an equilateral triangle, and $OPEN$ is a square. Determine the size of $\angle EAR$.



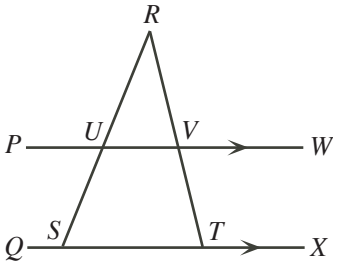
17. A beam of light shines from point S, reflects off a reflector at point P, and reaches point T so that PT is perpendicular to RS. What is the value of x?



18. In the diagram, let M be the point of intersection of the three altitudes of triangle ABC. If $AB = CM$, then what is $\angle BCA$ in degrees?



19. In the diagram, PW is parallel to QX, S and T lie on QX, and U and V are the points of intersection of PW with SR and TR, respectively. If $\angle SUV = 120^\circ$ and $\angle VTX = 112^\circ$, what is the measure of $\angle URV$?

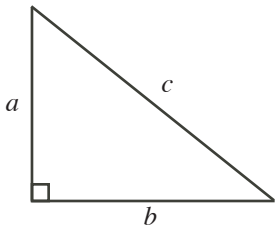


20. Three regular polygons meet at a point and do not overlap. One has 3 sides and one has 42 sides. How many sides does the third polygon have? Can you find other sets of three polygons that have this property?

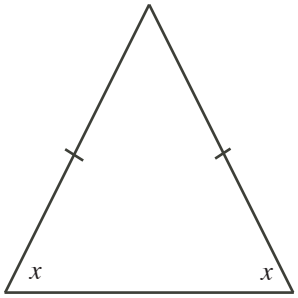
2. Seven Facts About Side Lengths

SF1. In a right-angled triangle, the Pythagorean Theorem tells us that $a^2 + b^2 = c^2$.
Can you prove this?

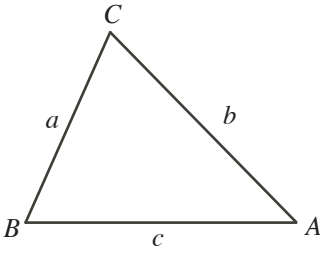
A *Pythagorean Triple* is a triple (a, b, c) of positive integers with $a^2 + b^2 = c^2$. What Pythagorean Triples do you know?



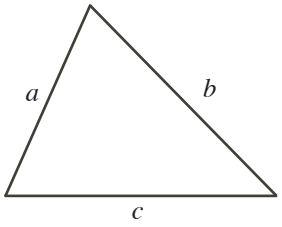
SF2. If a triangle has two angles equal, then the two opposite sides are equal.



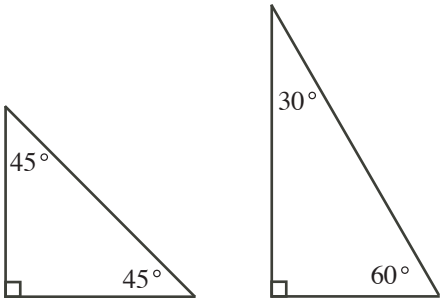
SF3. If $\angle A < \angle B$, then $a < b$.
If $a < b$, then $\angle A < \angle B$.



SF4. If a, b and c are the side lengths of a triangle, the Triangle Inequality tells us that $b + c > a$ and $a + c > b$ and $a + b > c$.
Can you explain why this is true?



SF5. There are two kinds of special triangles.
The first has angles 45°, 45° and 90°.
The second has angles 30°, 60° and 90°.
If the shortest side in each has length 1, what are the other side lengths?
These can be scaled by any factor.



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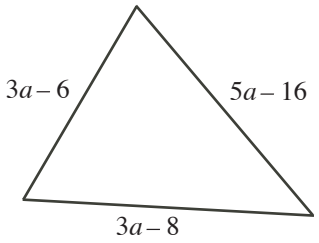
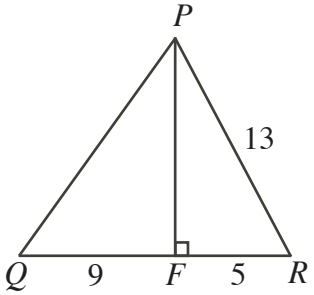
3. Intermediate Week 2 Problem Set 1

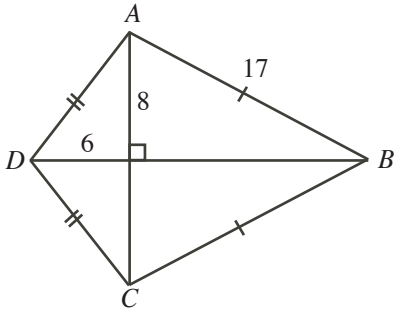
1. Determine the number of different values of a for which the given triangle is isosceles.



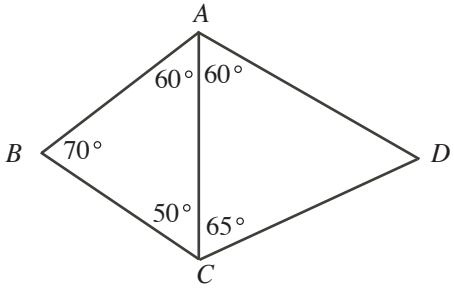
2. In triangle PQR, F is the point on QR so that PF is perpendicular to QR. If $PR = 13$, $RF = 5$, and $FQ = 9$, what is the perimeter of $\triangle PQR$?



3. Calculate the area of figure $ABCD$.



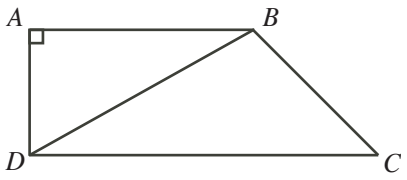
4. In the diagram, which side is the longest: AB, BC, AC, CD, or AD?



5. If a 3 m stake casts a shadow 7 m long, what is the height of a tree that casts a shadow 63 m long?

6. A *scalene* triangle is a triangle whose side lengths are all different. Determine the side lengths of all possible scalene triangles with integer side lengths and perimeter less than 13.

7. In the diagram, $AB = 4$, $DC = 6$, and AB is parallel to DC. If $\angle C = 45°$, determine the length of BD.

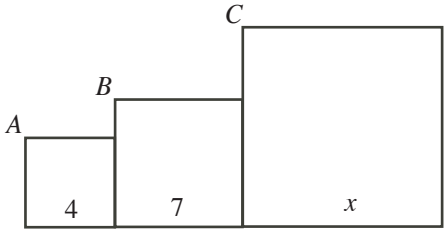


8. Triangle ABC is similar to $\triangle XYZ$. If $AB = 4$, $YZ = 9$, and $BC = XY = p$, determine the value of p.

9. A triangle can be formed having side lengths 4, 5 and 8. It is impossible however, to construct a triangle with side lengths 4, 5 and 10. Using the side lengths 2, 3, 5, 7 and 11, how many different triangles *with exactly two equal sides* can be formed?

10. A triangle can be formed having side lengths 4, 5 and 8. It is impossible however, to construct a triangle with side lengths 4, 5 and 10. Ron has eight sticks, each having an integer length. He observes that he cannot form a triangle using any three of these sticks as side lengths. What is the shortest possible length of the longest of the eight sticks?

11. In the adjacent squares shown, the vertices A, B and C lie in a straight line. What is the value of x?



12. In the diagram, $AD = BD = 5$, $EC = 8$ and $AE = 4$. Determine the length of BC.