



## Intermediate Math Circles for Wednesday 06 October 2010

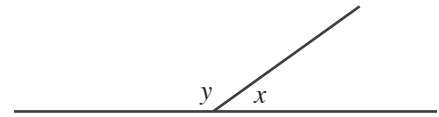
### 1. Opening Problem

One coin among  $N$  identical-looking coins is a fake and is slightly heavier than the others, which all have the same weight. To compare two groups of coins, you are allowed to use a set of scales with two pans which balance exactly when the weight in each pan is the same. Dan has to find the fake coin using at most two such comparisons.

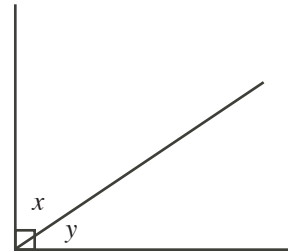
- (a) Explain how Dan can find the fake when  $N = 4$ .
- (b) Explain how Dan can find the fake when  $N = 6$ .
- (c) Explain how Dan can find the fake when  $N = 8$ .
- (d) Can Dan find the fake when  $N = 9$ ?
- (e) Can Dan find the fake when  $N = 10$ ?

## 2. Ten Facts About Angles

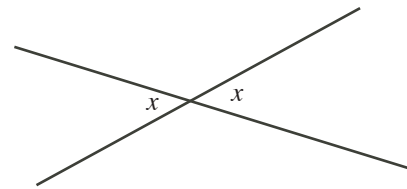
AF1. Angles along a straight line add to  $180^\circ$ .  
They are called *supplementary* angles.



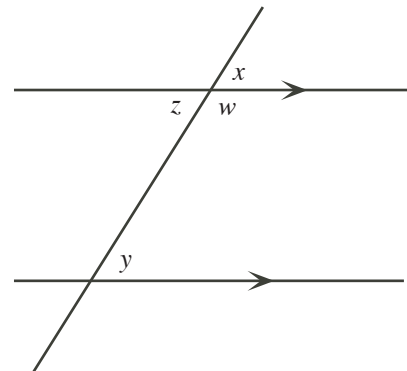
AF2. Angles in a right angle add to  $90^\circ$ .  
They are called *complementary* angles.



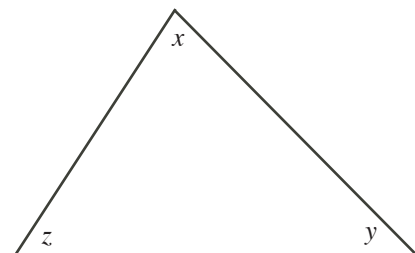
AF3. Opposite angles are equal. Why?  
Angles around a point add to  $360^\circ$ . Why?



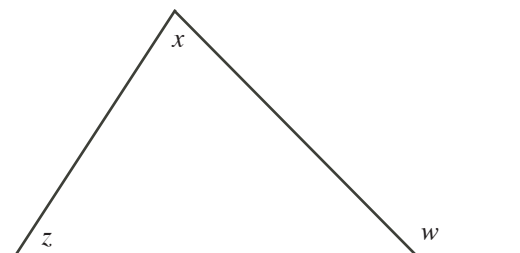
AF4. Consider two parallel lines.  
Corresponding angles are equal ( $x = y$ ).  
Alternating angles are equal ( $y = z$ ).  
Interior angles are supplementary ( $w + y = 180^\circ$ ).



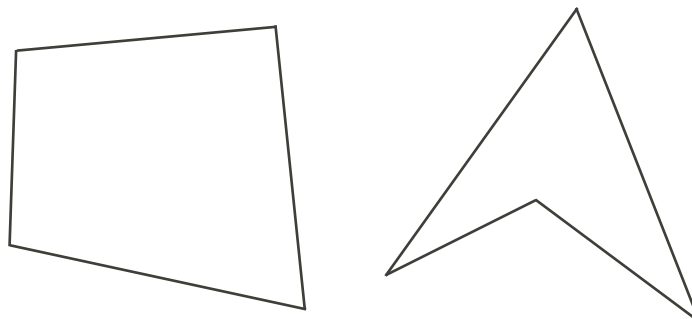
AF5. Angles in a triangle add to  $180^\circ$ .  
Can you prove this?



AF6. The exterior angle of a triangle equals the sum of the other two interior angles. That is,  $w = x + z$ .



AF7. What is the sum of the angles in a quadrilateral?  
How could we figure this out?



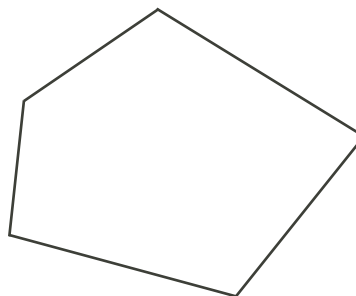
AF8. What is the sum of the angles in a pentagon?  
In a hexagon?  
In a polygon with  $n$  sides?

A *regular* polygon has all angles equal and all side lengths equal.

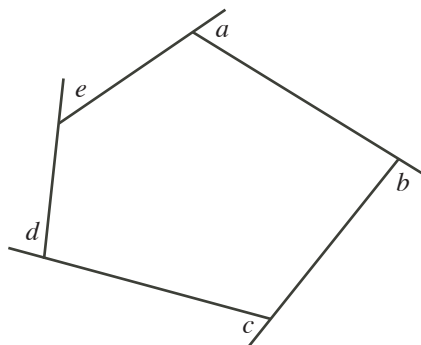
What is each angle in a regular pentagon?

What is each angle in a regular hexagon?

What is each angle in a regular decagon?

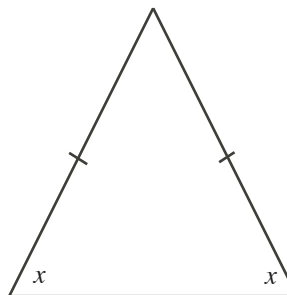


AF9. The sum of the exterior angles in a pentagon is  $360^\circ$ .  
Can you find two different ways to show this?  
The sum of the exterior angles in any convex polygon is  $360^\circ$ .



AF10. A triangle is called *isosceles* if two of its side lengths are equal and is called *equilateral* if all three side lengths are equal.

If a triangle has two side lengths equal, then the opposite two angles are equal.

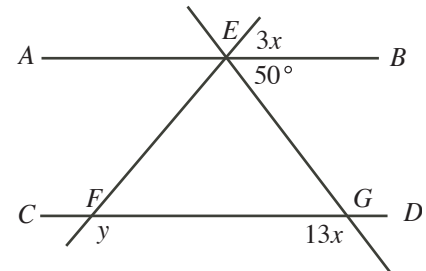




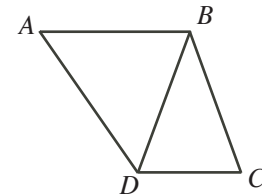
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### 3. Intermediate Week 1 Problem Set 1

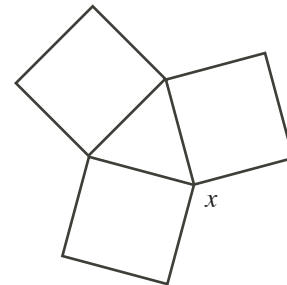
1. In the diagram,  $AB$  is parallel to  $CD$ . Determine the values of  $x$  and  $y$ .



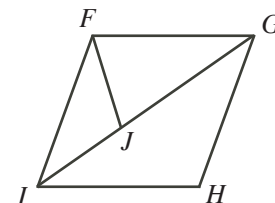
2. Triangle  $ABC$  has a right angle at  $B$ .  $AC$  is extended to  $D$  so that  $CD = CB$ . The bisector of angle  $A$  meets  $BD$  at  $E$ . Prove that  $\angle AEB = 45^\circ$ .
3. In the diagram,  $AB$  is parallel to  $DC$  and  $AB = BD = BC$ . If  $\angle A = 52^\circ$ , determine the measure of  $\angle DBC$ .



4. The diagram shows three squares of the same size. What is the value of  $x$ ?

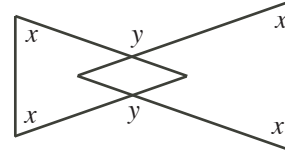


5. The diagram shows a rhombus  $FGHI$  and an isosceles triangle  $FGJ$  in which  $GF = GJ$ . Angle  $FJI$  equals  $111^\circ$ . What is the measure of angle  $JFI$ ?

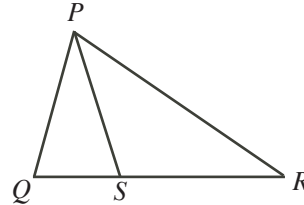


6.  $ABCD$  is a square. The point  $E$  is outside the square so that  $CDE$  is an equilateral triangle. Find angle  $BED$ .

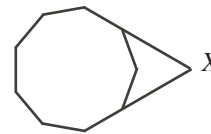
7. The diagram shows two isosceles triangles in which the four angles marked  $x$  are equal. The two angles marked  $y$  are also equal. Find an equation relating  $x$  and  $y$ .



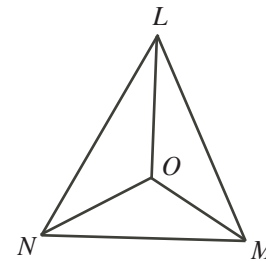
8. In the diagram,  $QSR$  is a straight line.  $\angle QPS = 12^\circ$  and  $PQ = PS = RS$ . What is the size of  $\angle QPR$ ?



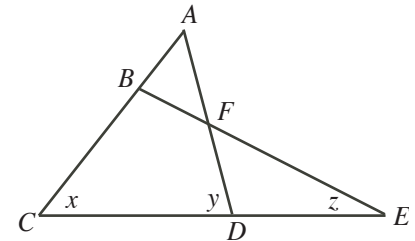
9. The diagram shows a regular nonagon with two sides extended to meet at point  $X$ . What is the size of the acute angle at  $X$ ?



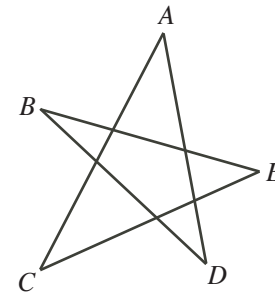
10. The three angle bisectors of triangle  $LMN$  meet at a point  $O$  as shown. Angle  $LMN$  is  $68^\circ$ . What is the size of angle  $LOM$ ?



11. In the figure shown,  $AB = AF$  and  $ABC$ ,  $AFD$ ,  $BFE$ , and  $CDE$  are all straight lines. Determine an equation relating  $x$ ,  $y$  and  $z$ .

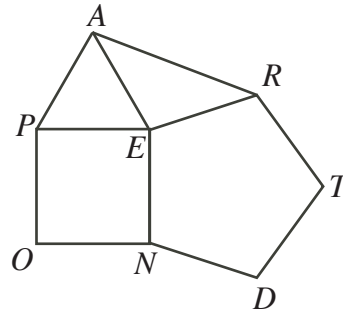


12. The angles of a nonagon are nine consecutive numbers. What are these numbers?
13. What is the measure of the angle formed by the hands of a clock at 9:10?
14. Determine the sum of the angles  $A$ ,  $B$ ,  $C$ ,  $D$ , and  $E$  in the five-pointed star shown.

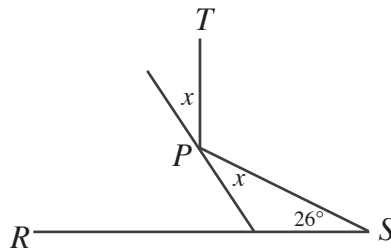


15. In  $\triangle PQR$ ,  $PQ = PR$ .  $PQ$  is extended to  $S$  so that  $QS = QR$ . Prove that  $\angle PRS = 3(\angle QSR)$ .

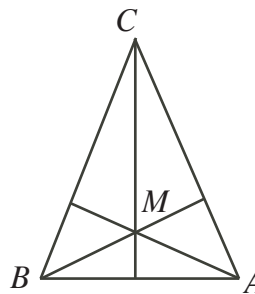
16. A regular pentagon is a five-sided figure which has all of its angles equal and all of its side lengths equal. In the diagram,  $TREND$  is a regular pentagon,  $PEA$  is an equilateral triangle, and  $OPEN$  is a square. Determine the size of  $\angle EAR$ .



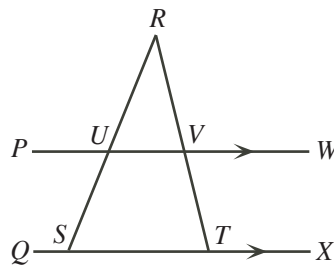
17. A beam of light shines from point  $S$ , reflects off a reflector at point  $P$ , and reaches point  $T$  so that  $PT$  is perpendicular to  $RS$ . What is the value of  $x$ ?



18. In the diagram, let  $M$  be the point of intersection of the three altitudes of triangle  $ABC$ . If  $AB = CM$ , then what is  $\angle BCA$  in degrees?



19. In the diagram,  $PW$  is parallel to  $QX$ ,  $S$  and  $T$  lie on  $QX$ , and  $U$  and  $V$  are the points of intersection of  $PW$  with  $SR$  and  $TR$ , respectively. If  $\angle SUV = 120^\circ$  and  $\angle VTX = 112^\circ$ , what is the measure of  $\angle URV$ ?



20. Three regular polygons meet at a point and do not overlap. One has 3 sides and one has 42 sides. How many sides does the third polygon have? Can you find other sets of three polygons that have this property?