University of Waterloo
Faculty of Mathematics

Centre for Education in
Mathematics and Computing

Grade 7 & 8 Math Circles
October 13, 2010
Percents and Ratios

Definition
Percent: a way of expressing a number as a fraction of 100.

ie. $\frac{53}{100} = 53\%$

Converting

1. **Decimals to Percentages**

   (a) Multiply the decimal number by 100
   OR

   (b) Move the decimal point two places to the right

   NOTE: don't forget to add the percent sign.
   Example

Decimal		Percent
0.75	(0.75)(100)	75%
0.75	0.75	75%

2. **Percentages to Decimals**

   (a) Divide the number by 100.
   OR

   (b) Move the decimal point two places to the left.

   NOTE: don't forget to remove the percent sign.
   Example

Percent		Decimal
75%	$\frac{75}{100}$	0.75
75%	75.0	0.75

3. Fractions to Percentages

(a) Divide the numerator by the denominator.

(b) Multiply by 100.

NOTE: don't forget to add the percent sign.

Fraction		**Percent**
$\frac{7}{8}$	$\frac{7}{8}(100)$	87.5%

Exercise 1

1. Convert the following to percentages. If needed, round your answer to the nearest hundredth.
   (a) 0.28 (b) $\frac{7}{9}$ (c) 0.536
2. Convert the following percentages to decimals:
   (a) 34% (b) 63.91% (c) 125%

Percentage Difference

Percentage difference is used to compare a new value to an original value. For example, what is the percentage difference when a T.V increases from $250 to $300?

Steps:

1. Calculate the increase or decrease.
2. Divide the difference by the original value.
3. Multiply by 100 to determine the percentage.

Formula:

$$PercentIncrease/Decrease = \frac{|AmountofIncrease/Decrease|}{OriginalAmount} \text{ x } 100$$

Example

What is the percentage difference when a T.V increases from $250 to $300?

Exercise 2

1. What is the percent difference if a ticket to Canada's wonderland used to be $34 and has changed to $52? Round your answer to the nearest hundredth.

2. Joey and Steve played in a basketball tournament. Joey made 24 baskets the first game and Steve made 12 baskets. They have decided that the player who has the lowest increase in his shot percentage will buy the other pizza. At the end of the second game Joey made 30 baskets and Steve made 18 baskets. Who will be buying the pizza?

3. A bakery has received feedback from their customers that there should be more chocolate chips in their cookies. If the bakery currently uses 2 bags of chocolate chips (each containing 150 chocolate chips) and they decide to make a 20% increase in the amount of chocolate chips used, what is the new amount of chocolate chips needed?

Mixing Problems
We can apply percentages to real life scenarios in determining the concentration of solutions and determining the quantities of different objects or substances needed.

When solving these problems it is easiest to organize the information in a chart. This allows you to clearly see what information you have and need.

Formula:

$$\left(\begin{matrix}\text{Final Solution}\\ \text{Amount}\end{matrix}\right)\left(\begin{matrix}\text{Percent of}\\ \text{Final Solution}\end{matrix}\right)=\left(\begin{matrix}\text{Amount of}\\ \text{Solution 1}\end{matrix}\right)\left(\begin{matrix}\text{Percent of}\\ \text{Solution 1}\end{matrix}\right)+\left(\begin{matrix}\text{Amount of}\\ \text{Solution 2}\end{matrix}\right)\left(\begin{matrix}\text{Percent of}\\ \text{Solution 2}\end{matrix}\right)$$

Exercise 3

1. Gary makes a 100mL solution in chemistry. He adds 25mL of a 45% acidic solution and 75mL of a 37% acidic solution. How acidic is the final solution?

2. Jolene is creating 2050mL of fruit punch for her friend's birthday. She adds 1000mL of apple juice that is 84% concentrated. Jolene also adds 1050mL of orange juice. If the fruit punch has a concentration of 88% what is the concentration of the orange juice? Round your answer to the nearest hundredth.

Ratios

Definitions
Ratio: a **ratio** is a comparison of quantities with the same units. **Ratios** can be expressed in ratio or fractional form.

$1:3$ or $\frac{1}{3}$('for every one pink ball there are 3 blue balls')

Proportion: a **proportion** is an equation stating two ratios are equivalent written in ratio or fractional form.

$$a:b=c:d \textbf{ OR } \frac{a}{b}=\frac{c}{d}$$
'a is to b as c is to d.'

So, if one ratio can be multiplied by a constant to obtain the second ratio, the two ratios are **proportional**.

Example

$$1 : 2 = 3 : 6 \text{ since } 3\text{x}(1 : 2) = 3 : 6$$

HINT: if the cross products are equal the ratios are proportional.

Exercise 4

1. Which of the following ratios are proportional?
   (a) $6 : 5$ and $18 : 15$ (b) $7 : 15$ and $24 : 30$ (c) $\frac{3}{8}$ and $\frac{18}{48}$ (d) $9 : 11$ and $63 : 77$
2. For the following, determine the value of x such that the two ratios are proportional:
   (a) $3 : 4 = x : 28$ (b) $11 : 12 = 121 : x$ (c) $\frac{x}{9} = \frac{15}{27}$
3. In April, 18 students bought chocolate ice cream and 21 bought strawberry ice-cream from the cafeteria. In May, 12 students bought chocolate ice-cream and 14 bought strawberry. Comparing the ratio of students who bought chocolate ice cream to strawberry in April and May, is this proportional? If not, which month sold a higher ratio of chocolate to strawberry?
4. In a Karate class there are 24 students who have green belts and 32 students who have blue belts among the 12 year olds. Among the 13 year olds, 18 students have green belts and 36 have blue belts. Which age group has a lower ratio of green belts to blue belts?

Problem Set

1. What fraction of the original square is shaded? 

2. Erica buys a pair of blue jeans for 25.95. If tax is 13% what must Erica pay?

3. Ben wrote 5 tests this year. He scored $\frac{9}{10}$, $\frac{46}{50}$, $\frac{21}{25}$, $\frac{4}{5}$, and 96%. What test score is the best?

4. Brookville school collected 6000 cans for their food drive last year. This year the school collected 3% fewer cans. How many cans did they collect this year?

5. Kate is selling lemonade to raise money for her soccer team. She is selling a cup of lemonade for $2.50 each. After three hours of selling lemonade she has only sold 4 cups of lemonade. Kate believes she is not selling her lemonade because the price is too high. She decides to reduce her price by 40%. What is the new cost for a cup of lemonade?

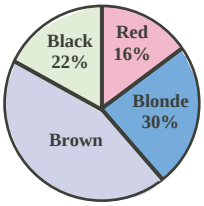
6. The dimensions of a 20 cm by 25 cm photo is enlarged to a 25 cm by 30 cm photo. What is the area's percentage of increase?

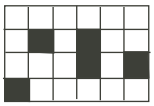
7. A mountain bike originally costs $750 and is reduced by 20%. Two weeks later the bike was increased by 20%. What is the final cost of the bike excluding taxes?

8. $\frac{3}{4}$ of a number is 24. What is $\frac{4}{5}$ of the same number?

9. For the Maple Leafs to make the playoffs they must win 60% of their games. If they have won 20 and lost 25 thus far how many games must they win out of the remaining 30 games?

10. The circle graph displays a survey that was completed on hair colour. If 600 people were surveyed how many people have brown hair?



11. How many additional squares must be shaded so the number of shaded squares is half the number of unshaded squares? 

12. In a class of 25 students, the ratio of boys to girls is 2:3. If 3 girls join the class what is the ratio of girls to boys in the class?

13. Andrew is given two 12cm long pieces of string. With one piece Andrew forms a square. With the other piece Andrew forms a rectangle whose width is double its length. Find the ratio of the area of the rectangle to the area of the square.

14. On Stacy's farm, every two horses share a trough, every three cows share a trough, and every eight pigs share a trough. Stacy has the same number of each animal, and she has a total of 69 troughs. How many animals does Stacy have on her farm?

15. If $a : b = 3 : 4$ and $a : (b + c) = 2 : 5$, find the value of a:c.

16. Gabe has an income which is five-eights of Heidi's. Gabe's expenses are one-half of Heidi's, and Gabe saves 40% of his income. What percentage of income does Heidi save?

Problem Set Solutions

1. $\frac{1}{4}$
2. 29.32
3. 96%
4. 5820
5. \$1.50
6. 50%
7. \$720
8. 25.6
9. 25
10. 192
11. 3
12. 5 : 9
13. 8 : 9
14. 216
15. 6 : 7
16. 25%