Exercise Solutions

Exercise 1.1

a) $2+4+6+8+\cdots+100$ b) $0+1+8+27+64+...+1000000$

Exercise 1.2

a) $\sum_{i=0}^{99} i^2$ b) $\sum_{j=0}^{\frac{n-1}{2}} (1+2k)$

Exercise 2.1

Mean: $\dfrac{15+45+36+24+68+...+67+62+50}{21} = \dfrac{907}{21} = 43.19$

Median: Writing the data in ascending order, we have

15 16 24 25 29 36 36 37 37 41 42 45 50 52 52 52 60 61 62 67 68

The median (number in the middle) is 42.

Mode: The mode is 52 because it appeared the most (3 times) out of the set of data.

Exercise 3.1

Arranging the data in ascending order, we have

10 12 19 24 25 31 36 37 38 41 45 64 65 74 74 75 82 85 87 93 96 97 98

The lower quartile is the in the $\frac{23+1}{4} = 6^{\text{th}}$ spot, which would be 31.

The upper quartile is the in the $\frac{3(23+1)}{4} = 18^{\text{th}}$ spot, which would be 85.

The interquartile range is $85 - 31 = 54$

Exercise 3.2

Arranging the data in ascending order, we have

4 6 10 10 12 13 14 15 16 19 27 29 36 37 40 46 54 70 72 72 73 78 78 79 85 87 92 92 95 96 98

The lower quartile is the in the $\frac{31+1}{4} = 8^{\text{th}}$ spot, which would be 15.

The upper quartile is the in the $\frac{3(31+1)}{4} = 24^{\text{th}}$ spot, which would be 79.

The interquartile range is $79 - 15 = 64$

Problem Set Solutions

1. $\sum_{j=0}^{50} (-1)^j (2j)$

2. $2 + 9 + 16 + 25 + 36 + 49 + 664 + 81 + 100$

3. The highest number in the set (excluding x) is 20, so x could be 9 to get a range of 11. The lowest number in the set (excluding x is 9, so x could be 20 to get a range of 11. $\therefore 9 \le x \le 20$

4. Adding the 22 data points, the mean is equal to

$$\begin{aligned} \frac{905 + x}{22} &= 42 \\ 905 + x &= (42)(22) \\ 905 + x &= 924 \\ x &= 19 \end{aligned}$$

5. Let x be the fourth number

$$\begin{aligned} \frac{4 + 8 + 9 + x}{4} &= 28 && (1) \\ 21 + x &= 112 && (2) \\ x &= 91 && (3) \end{aligned}$$

6. Let a and b be the sum of the three numbers with a mean of 9 and two numbers with a mean of 4 respectively. We have

$$\begin{aligned} \frac{a}{3} &= 9 & \qquad \frac{b}{2} &= 4 \\ a &= 27 & b &= 8 \end{aligned}$$

So the mean of the 5 numbers is

$$\begin{aligned} \frac{a + b}{5} &= \frac{27 + 8}{5} \\ &= \frac{35}{5} \\ &= 7 \end{aligned}$$

7. The largest number in the set is a number that would make the mean 9 even the other

five numbers are as small as possible. Let x be the largest number.

$$\begin{aligned}
\frac{1+2+3+4+5+x}{6} &= 9 \\
\frac{15+x}{6} &= 9 \\
15+x &= 54 \\
x &= 39
\end{aligned}$$

8. Let x be the middle of the three integers. Then the integers we are looking for are $x-1, x,$ and $x+1$

$$\begin{aligned}
\frac{10+19+(x-1)+x+(x+1)}{5} &= 25 \\
29+3x &= 125 \\
3x &= 96 \\
x &= 32
\end{aligned}$$

$\therefore$ The three integers are 31, 32, and 33.

9. The mean of the set is about 7.8, which means that we need to remove either a 7 or a 6. We cannot remove the 6 because it is the lowest number, and we don't want to change the range. Thus, we should remove a 7.

10. $(28\% - 7\%)$ of $200 = (0.21)(200) = 42$ more people would prefer swimming to basketball.

11. (a) The speeds of the drivers (in m/min) are
 $\frac{1500}{20} = 75$ $\quad \frac{1500}{35} \doteq 42.9$ $\quad \frac{1500}{30} = 50$ $\quad \frac{1500}{40} = 37.5$ $\quad \frac{1500}{25} = 60$
 The mean is $\dfrac{75+42.9+50+37.5+60}{5} = 53.1$ m/min

 (b) As mentioned in part (a), the average speed of the driver that finished first is $\frac{1500}{20} =$ 75 m/min

12. Let a represent the height of the first candle, b represent the height of the second candle and h represent the height of the candle at the start. Then the heights of the first and second candles are $a = -\dfrac{h}{4}t + h$ and $b = -\dfrac{h}{3}t + h$, where t is the elapsed time. We want

to find t such that $a = 2b$.

$$\begin{aligned}
a &= 2b \\
-\frac{h}{4}t + h &= 2(-\frac{h}{3}t + h) \\
-\frac{h}{4}t + h &= -\frac{2h}{3}t + 2h) \\
\frac{2h}{3}t - \frac{h}{4}t &= h \\
\frac{5\cancel{h}}{12}t &= \cancel{h} \\
t &= \frac{12}{5} \\
&= 2.4 \text{ hours}
\end{aligned}$$

13. Let n be the number of integers in the set.

$$\begin{aligned}
\frac{477}{n} &= 53 \\
\frac{477}{53} &= n \\
n &= 9
\end{aligned}$$

So 108 plus eight other numbers gives us 477. If we make seven numbers as small as possible, then the last number would be as big as possible, which would be
$477 - 108 - 1 - 2 - 3 - 4 - 5 - 6 - 7 = 341$

14. One of Mike's marks is 50, and another is 94. We also know that she has two 76's because that is the mode. It is also the median, so if we arrange the marks in ascending order, we have $50, a, 76, 76, b, 94$, where a and b are the other two marks. The mean is 74, so we have

$$\begin{aligned}
\frac{50 + 76 + 76 + 94 + a + b}{6} &= 74 \\
296 + a + b &= 444 \\
a + b &= 148
\end{aligned}$$

With the restrictions that $50 < a < 76$ and $76 < b < 94$, we find that a could be any integer from 54 to 72, which means there are 19 different possibilities for her second lowest mark.

15. Let a be the total amount of money and n be the number of coins. Then we have

$$\begin{aligned}
\frac{a}{n} &= 17 \\
a &= 17n
\end{aligned} \qquad \text{and} \qquad \frac{a-1}{n-1} = 18$$

Combining these, we have

$$\begin{aligned}
\frac{17n-1}{n-1} &= 18 \\
17n-1 &= 18n-18 \\
n &= 17 \Rightarrow a = 17n = (17)(17) = 289
\end{aligned}$$

To get \$2.89 from 17 coins we must have 4 pennies, 2 nickels, 11 quarters and no dimes. $\therefore$ there are 2 nickels in the purse.

16. The median is 9 and 8 appears more than once, so if we arrange the 5 integers in ascending order, we have $8, 8, 9, a, b$ where a and b are the remaining integers. In order to find the largest possible integer, we make a as small as possible, so $a = 10$

$$\begin{aligned}
\frac{8+8+9+10+b}{5} &= 10 \\
35+b &= 50 \\
b &= 15
\end{aligned}$$